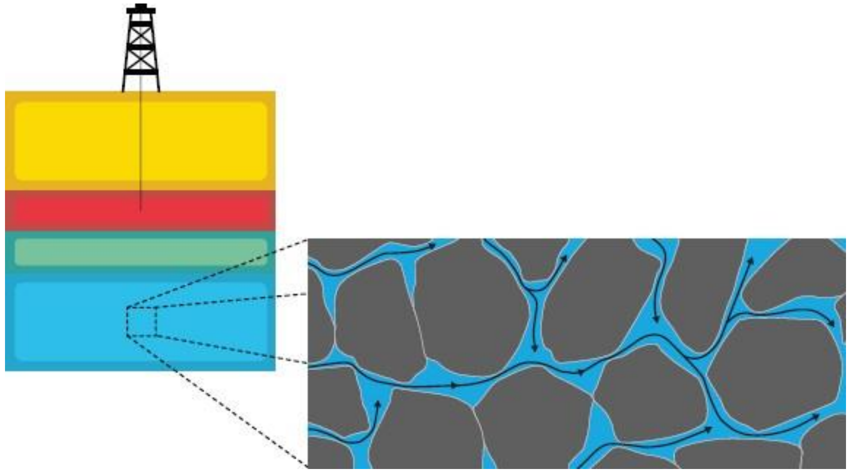
Petrophysics

PERMEABILITY

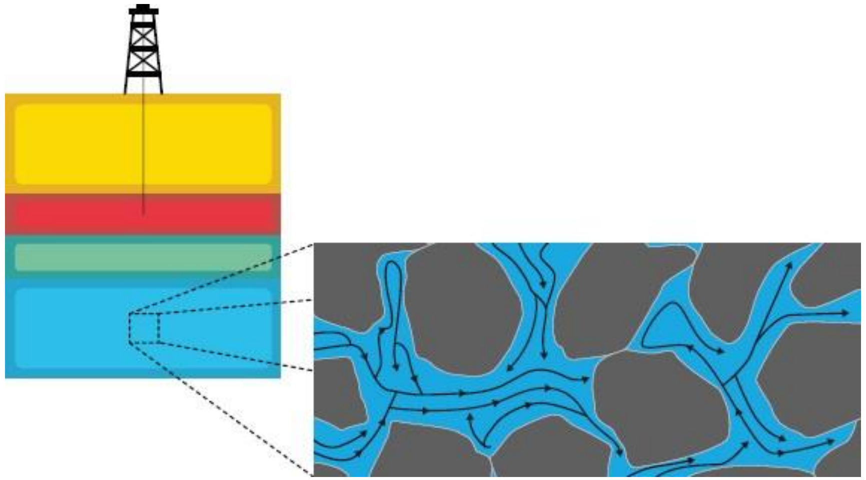
PERMEABILITY

Permeability

- It is a measure of the ease with which fluid flows in a porous medium.



Low permeability rock



High permeability rock

- When a rock is 100% saturated with a single phase (water, oil, or gas) the measured permeability is called “absolute permeability”, “single-phase permeability” or just ”Permeability”.

Permeability

- Permeability is a flow or transport property that helps in understanding the flow in reservoirs.
- Darcy's law:

$$q = -\frac{kA}{\mu L} dP$$

where,

q : the flow rate [m^3/s]

k : the permeability [m^2]

A : the core cross-sectional area [m^2]

μ : the viscosity of the fluid injected [Pa·s or $N/m^2 \cdot s$]

L : the length of the core [m]

dP : the pressure difference across the core [Pa or N/m^2]

Permeability

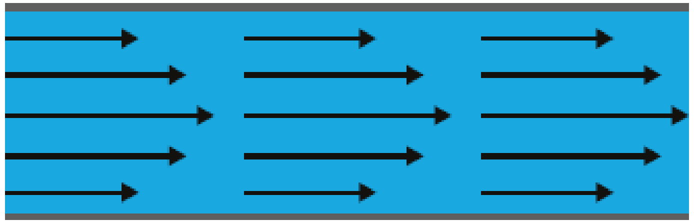
- **Applications of Permeability:**
  - Permeability describes the flow in porous media
  - From Darcy's law, the production flow rate from any given reservoir to the surface can be estimated.
  - The permeability of a given reservoir can be determined through lab experiments on core samples that are extracted from the same reservoir.

- **Validity of Darcy's law for Single-Phase Permeability:**
  - The core sample must be 100% saturated with a single phase
  - The flow has to be laminar
  - The flow has to be steady-state flow

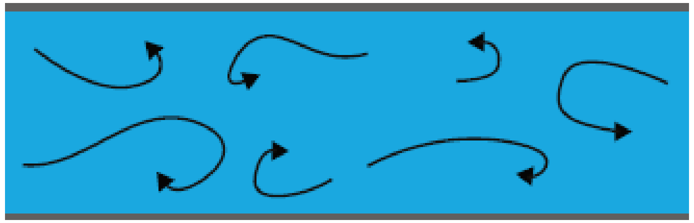
Validity of Darcy's law for Single-Phase Permeability

- The core sample must be 100% saturated with a single phase (water, oil, or gas)

- The flow has to be laminar:
  - slow, uniform flow



Laminar flow

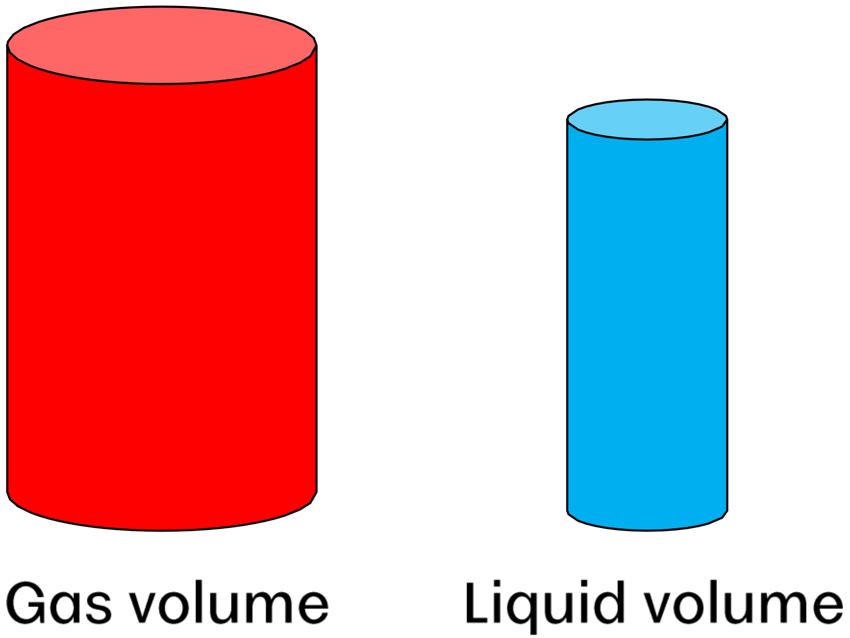


Turbulent flow

- The flow has to be steady-state flow:
  - No volumetric change over time
  - Darcy's law is invalid if the flow is unsteady-state

Darcy's law under different boundary conditions

- Fluids can either be:
  - Compressible (change volume due to change in pressure such as gases)
  - Incompressible (does not change volume due to pressure such as liquids)



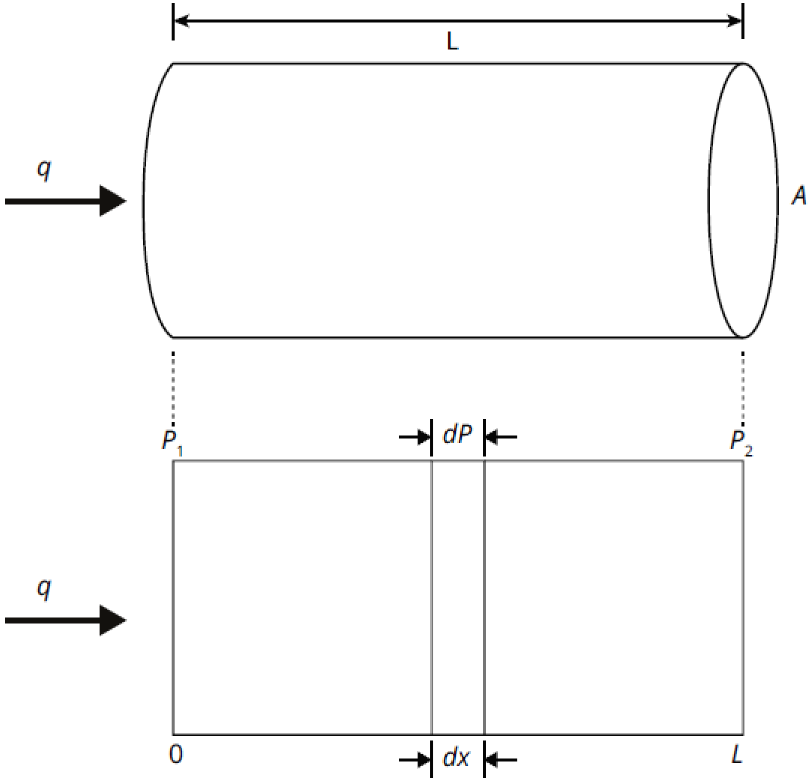
Linear solution for Darcy's law for incompressible fluids

$$q = \frac{kA}{\mu}\frac{dP}{dx}$$

$$q dx = \frac{kA}{\mu} dP$$

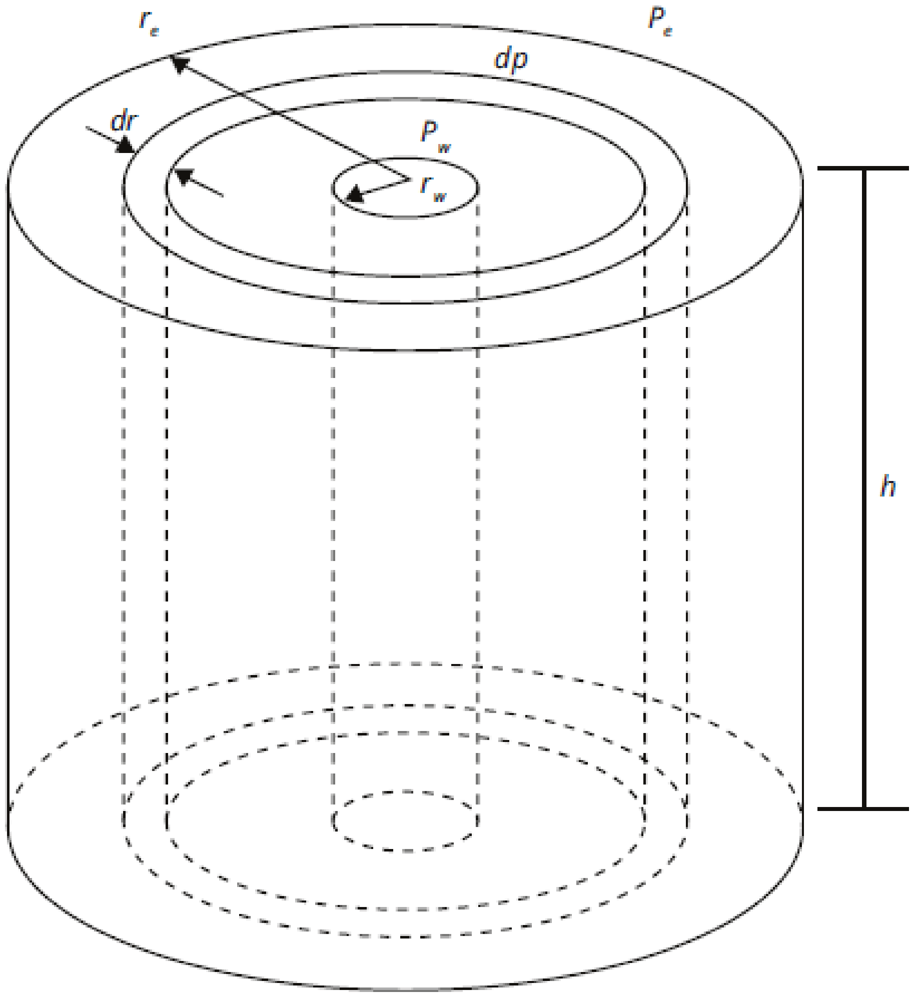
$$q \int_0^L dx = \frac{kA}{\mu}\int_{P_2}^{P_1} dP$$

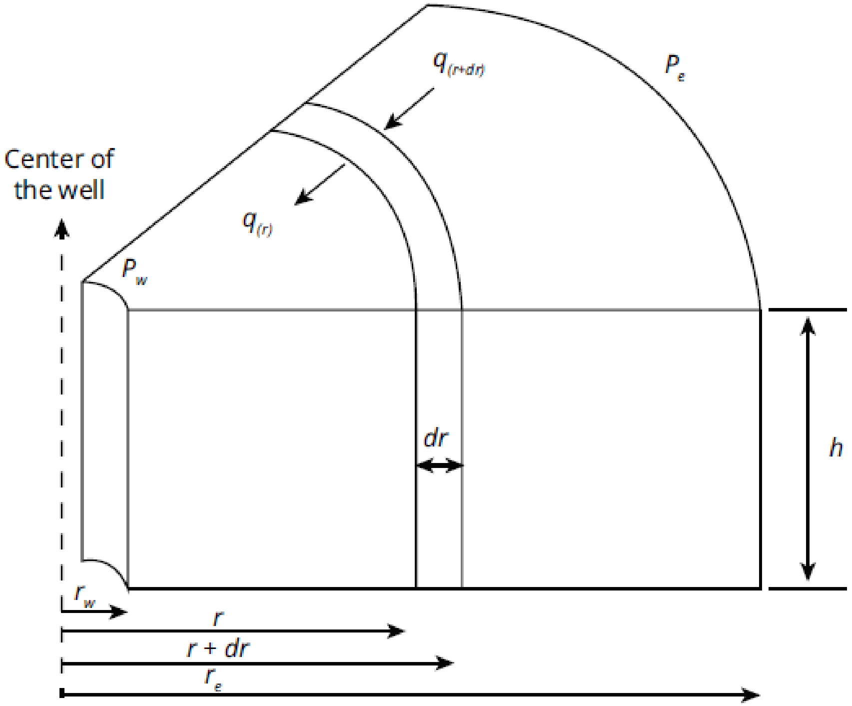
$$q(L-0) = \frac{kA}{\mu}(P_1 - P_2)$$



$$q = \frac{kA}{\mu L}(P_1 - P_2)$$

Steady-state Radial Solution of Darcy's law for Incompressible Fluids





$$q = \frac{2\pi kh\,(P_e - P_{wf})}{\mu ln\,\frac{r_e}{r_w}}$$

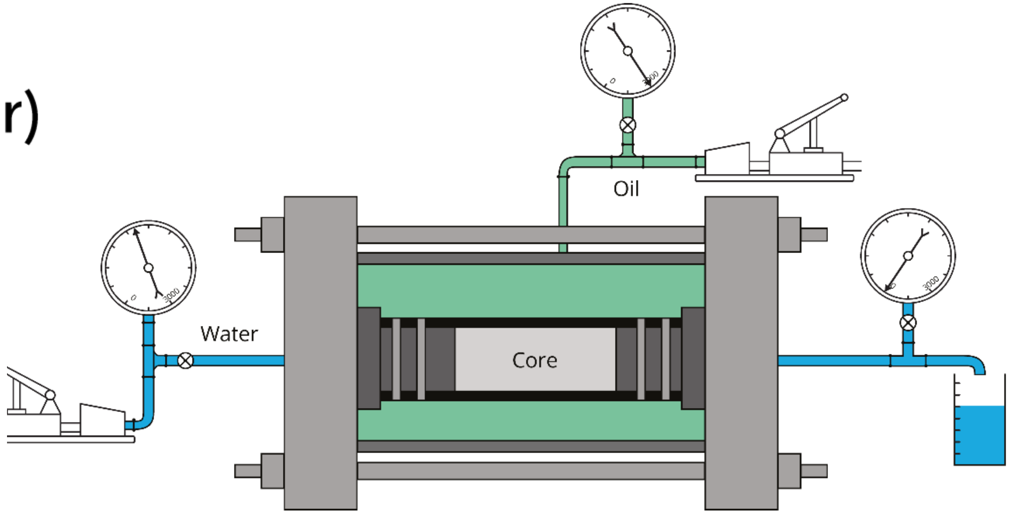
Permeability – Unit Systems

Parameter	SI Units	Darcy Units	Oilfield Units
q (flow rate)	[m^3/s]	[cm^3/s] or [cc/s]	[bbl/d]
L (length)	[m]	[cm]	[ft]
A (area)	[m^2]	[cm^2]	[ft^2]
h (thickness)	[m]	[cm]	[ft]
r (radius)	[m]	[cm]	[ft]
P (pressure)	[Pa]	[atm]	[psia]
k (permeability)	[m^2]	[D]	[mD]
μ (viscosity)	[Pa.s]	[cP]	[cP]

Laboratory Measurements

- The permeability of a core sample is measured using either liquid or gas.

1. Liquid Permeability
   - Measure the dimensions (length and diameter)
   - Vacuum the core prior to injecting liquid (to remove air and ensure a single-phase flow)
   - Apply confining pressure to ensure that the liquid to be injected will pass through the core sample and will not bypass it. Apply confining pressure = 1.5x the liquid injection pressure
   - Inject liquid (e.g. water) at a specific rate
   - Wait until the inlet and outlet pressures become constant and do not fluctuate (i.e. steady-state)



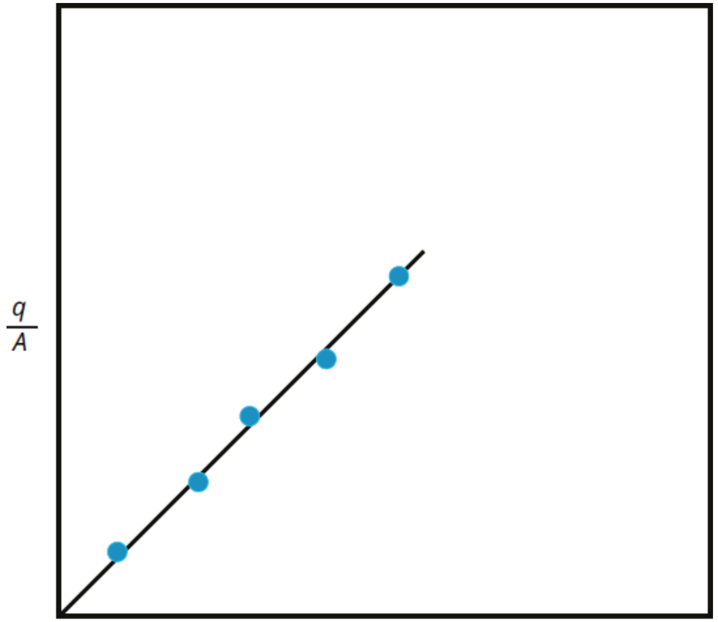
Laboratory Measurements

1. Liquid Permeability (continue)

- Vacuum the core prior to injecting liquid (to remove air and ensure a single-phase flow)
- Change the flow rate and follow the same procedure
- Record few such data points, then plot the data to find the permeability of the core sample

- Understanding the plot:
  - If we rearrange Darcy's law as follows: $\frac{q}{A} = \frac{k}{\mu}\frac{dP}{L}$
  - This is the linear form: $y = mx + b$

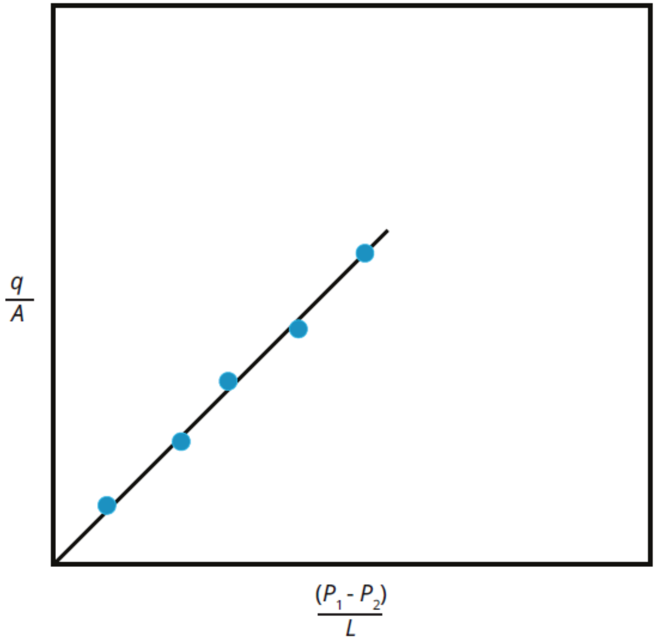
    where, $y = \frac{q}{A}$, $x = \frac{dP}{L}$, $m = \frac{k}{\mu}$
  - b is the y-intercept and here it is 0 since the intercept is the origin



Laboratory Measurements

1. Liquid Permeability (continue)

- Since the slope in this plot is $m = \frac{k}{\mu}$, to find the permeability, multiply the slope by the viscosity.
- When analyzing the experimental data, make sure you follow consistent units. In labs it is common to use Darcy's units (check the unit systems in the previous lecture)



$$m = \frac{k}{\mu}$$

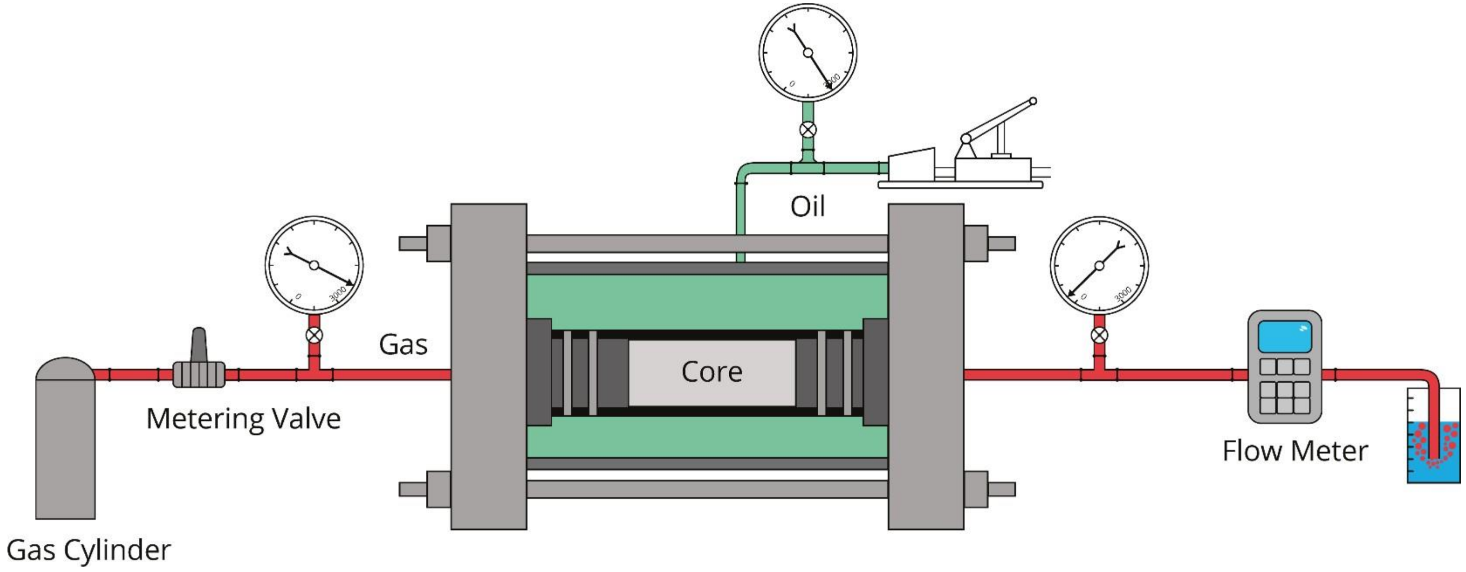
$$k = m * \mu$$

Laboratory Measurements

- Gas Permeability
  - Since gas is a compressible fluid, unlike the liquid case, the governing equation is going to change
  - Measuring gas permeability has advantages over measuring liquid perm:
    - It takes less time
    - Gas does not wet the core sample (the core can be reused for other analysis)
  - Measuring gas permeability has one disadvantage:
    - The gas permeability requires correction as it tends to be overestimated compared to liquid permeability

Laboratory Measurements

- Gas Permeability (continue)
  - The core holder is attached to a gas cylinder
  - A metering valve is used to vary the flow rate
  - A flow meter is used at the end of the core to measure the flow rate at atmospheric conditions



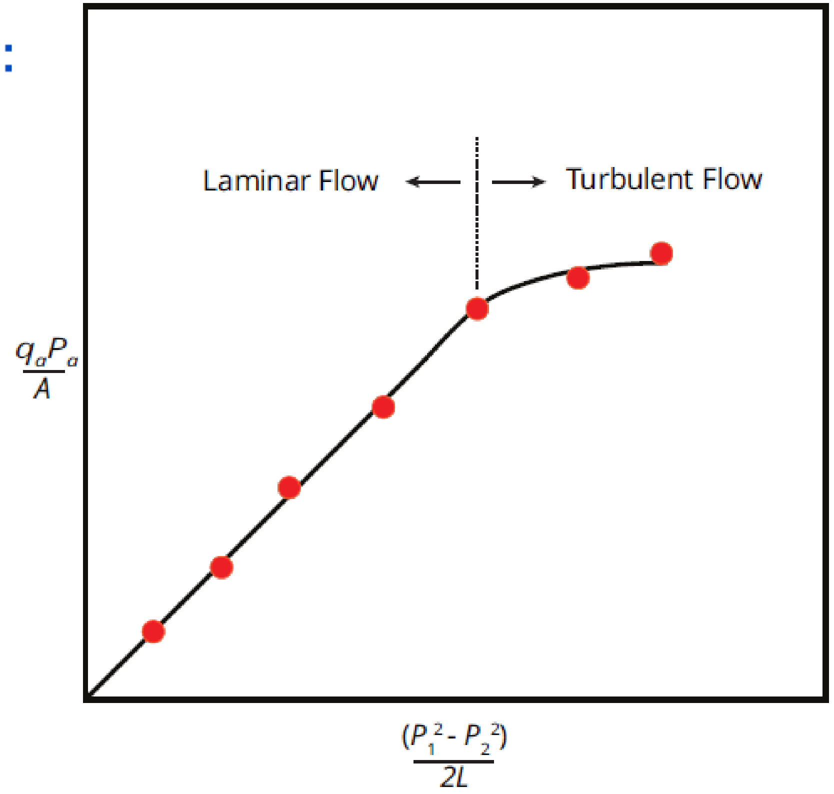
Laboratory Measurements

- Gas Permeability (continue)
  - Since the flow rate varies across the core, the use of the average pressure is more representative of the flow in the core
  - The governing equation, when using Darcy's units, for gas is:

$$q_a = \frac{kA}{2\mu L}(P_1^2 - P_2^2)$$

Atmospheric flow rate

  - Similar to liquid permeability, rearrange the equation to find the gas permeability across the core after acquiring several data points
  - Gases have a lower viscosity than liquids, therefore it is common to reach a higher flow rate than liquids. This can result in turbulent flow that makes Darcy's law invalid.
  - Turbulent flow data, should be omitted from the analysis.



Laboratory Measurements

- Gas Permeability (continue)

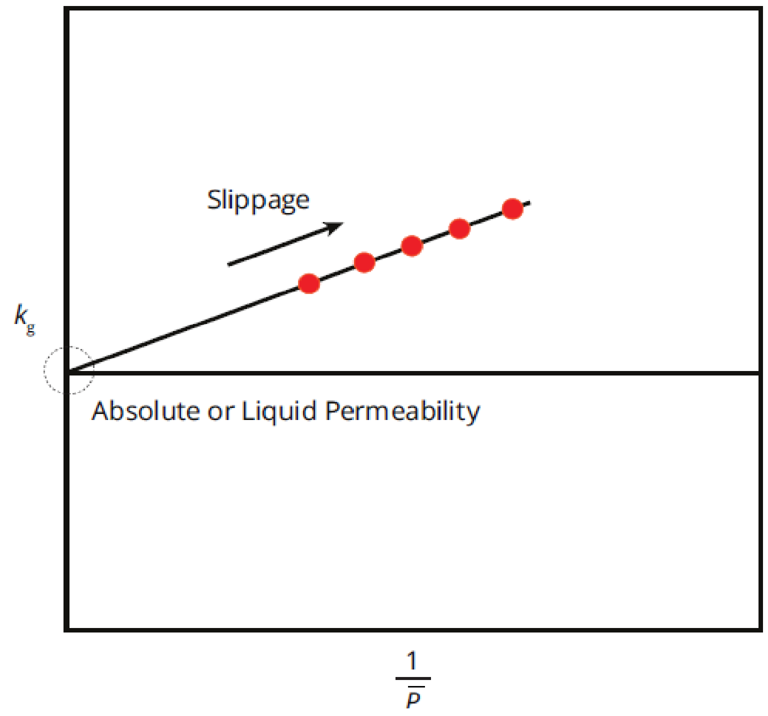
➢ Gas permeability tends to be higher than the liquid one. This is due to gas slippage at the pore wall (Klingenberg effect)

➢ This gas slip makes the permeability higher than what it should be, therefore not representative of the actual value

➢ This can be corrected by finding the equivalent liquid permeability. To do so:

- Compute the gas permeability (k_g) at every data point
- Plot these values against the inverse of the average pressure ($\bar{P}$)
- The y-intercept in this case is the liquid permeability (k_L)

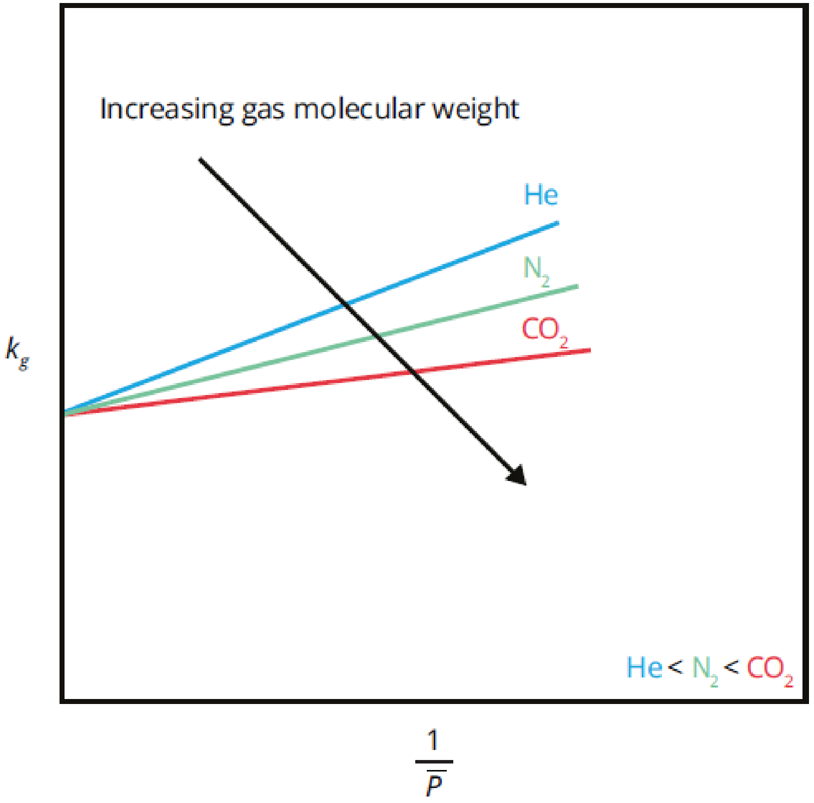
➢ At infinite pressure, gas can be considered to behave like a liquid



Laboratory Measurements

- Gas Permeability (continue)

➢ The molecular weight (MW) of gas affects the slippage. As the gas MW increases, the slippage decreases since gas becomes heavier and closer to liquid.



Pressure Profile

- Liquid Flow
  - By knowing the permeability, we can know the pressure at any point in the core by rearranging Darcy's law for liquids as follows:

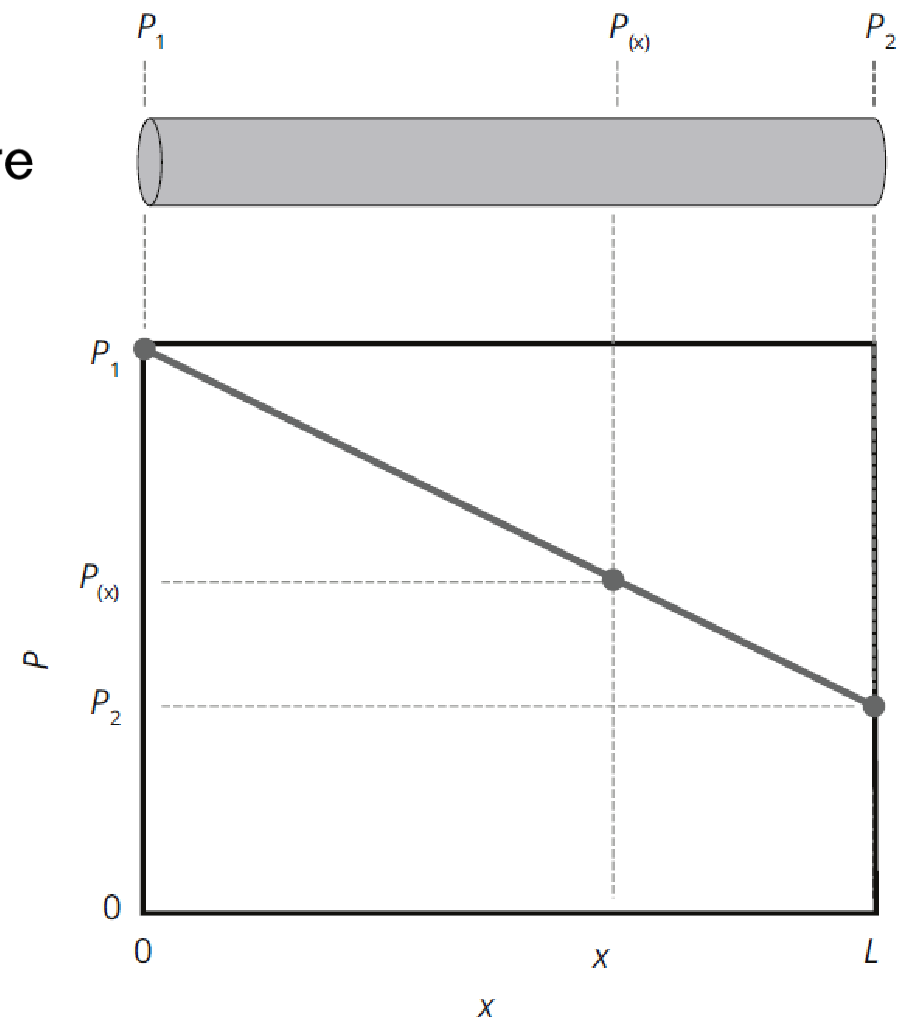
$$P(x) = P_1 - \frac{q\mu}{kA}x$$

  - The pressure profile is expected to be linear

At $x = 0$ the pressure $= P_1$

At x the pressure $= P_{(x)}$

At $x = L$ the pressure $= P_2$



Pressure Profile

- Gas Flow
  - Since dealing with gases is different than liquids, a form of Darcy's law that is suitable for gases will be used here:

$$P_{(x)}^2 = P_1^2 - \frac{2q_a\mu}{kA}x$$

  - Then,

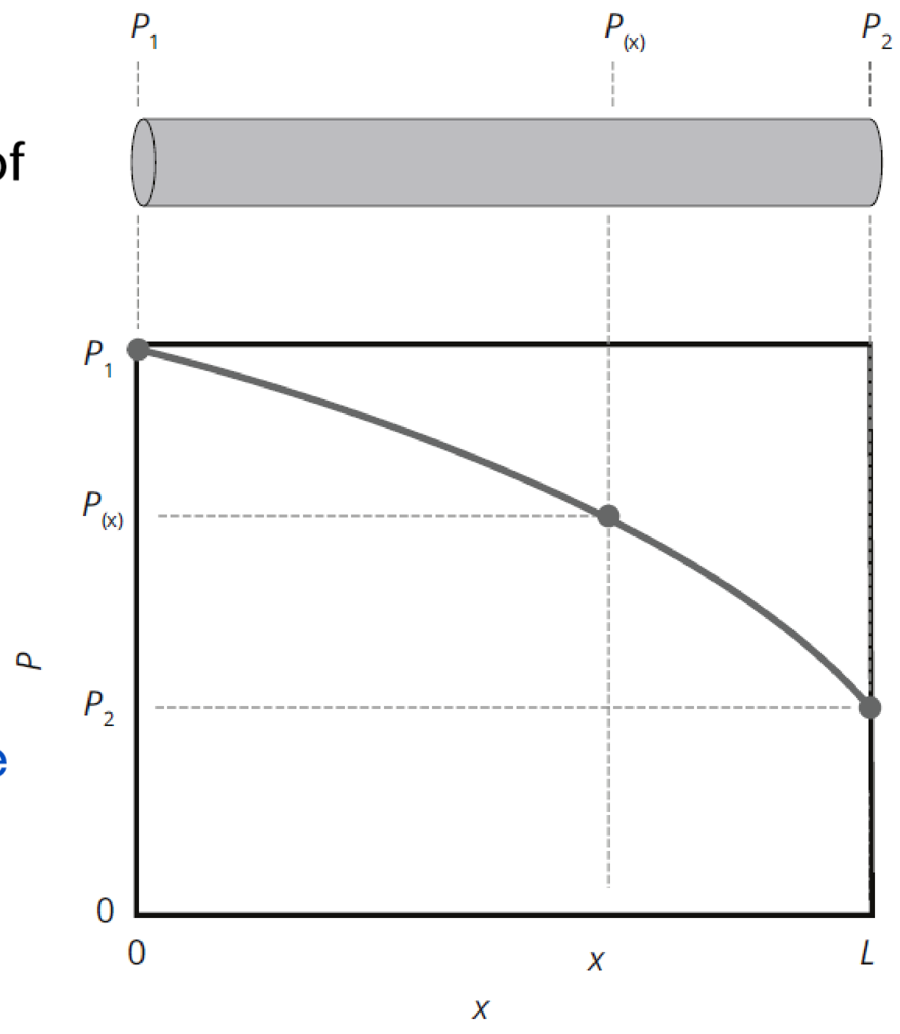
$$P(x) = \sqrt{P_1^2 - \frac{2q_a\mu}{kA}x}$$

  - The pressure profile is expected to have a parabolic shape

At $x = 0$ the pressure $= P_1$

At x the pressure $= P_{(x)}$

At $x = L$ the pressure $= P_2$



Pressure Unit Conversion

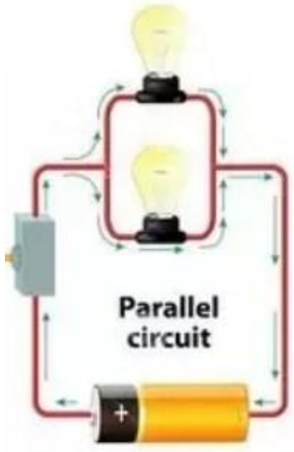
$$psia = psig + 14.7$$

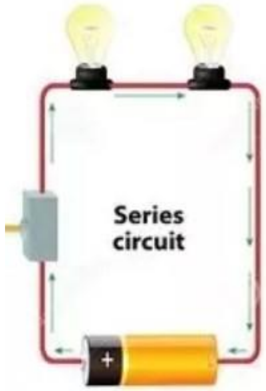
$$atm = \frac{psia}{14.7}$$

$$atm = \frac{psig}{14.7} + 1$$

Introduction to Layered Flow

- The aim of understanding the flow in layered systems having beddings with different permeabilities is to find the average permeability across the system
- The concept resembles electrical circuits
- The average permeability will vary based on the type of the beddings in the system (parallel or series)





Average Permeability

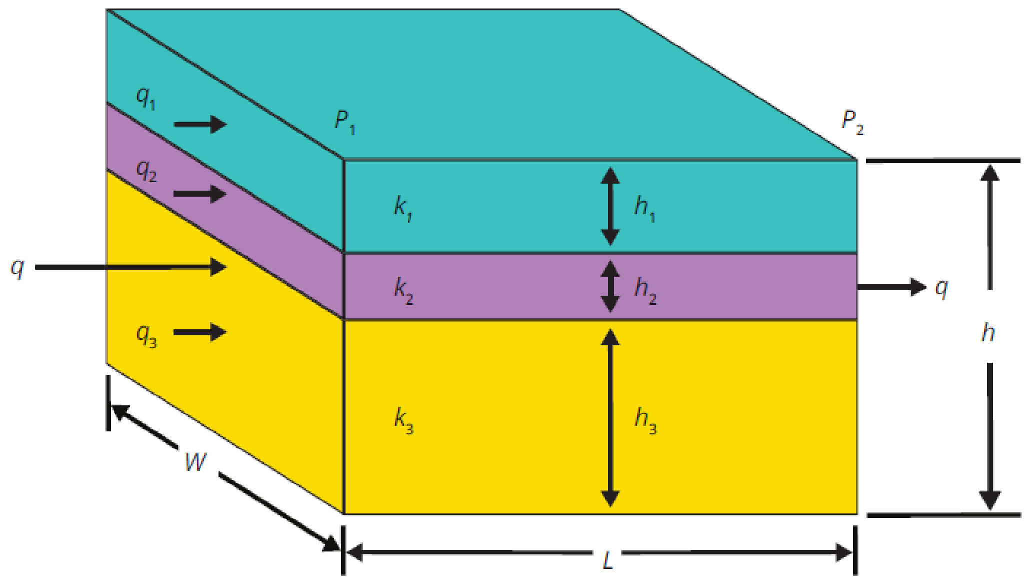
- **Linear Parallel System**
  - The pressure difference across the system is constant:

$$\Delta P_1 = \Delta P_2 = \Delta P_3$$

  - if:

$$k_1 \neq k_2 \neq k_3$$

  - Thus, the flow rate will be different across each layer as it is a function of the permeability, so the total flow rate is: $q = q_1 + q_2 + q_3$
  - It is also known that: $h = h_1 + h_2 + h_3$
  - Therefore, the total flow rate of the system is: $q = \dfrac{\bar{k} W h (P_1 - P_2)}{\mu L}$



Average Permeability

- **Linear Parallel System**
  - If we substitute the flow rate from Darcy's law in:

$$q = q_1 + q_2 + q_3$$

  - It becomes:

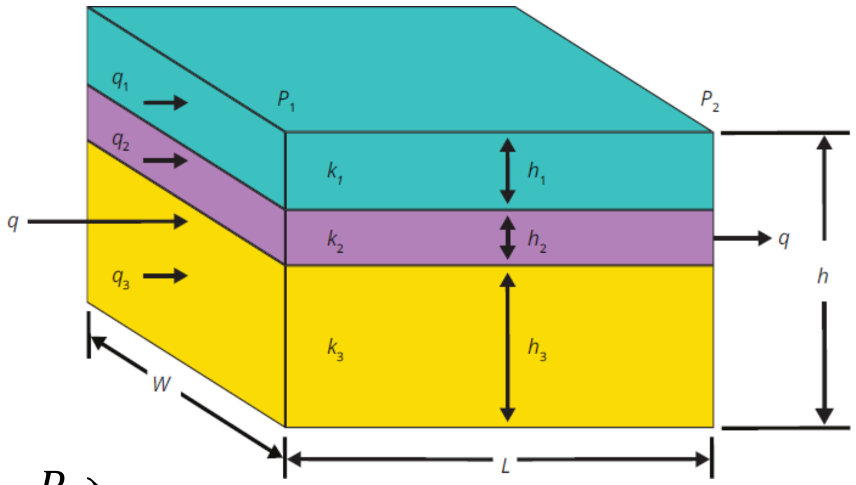
$$q = \frac{\bar{k}Wh(P_1 - P_2)}{\mu L} = \frac{k_1 W h_1 (P_1 - P_2)}{\mu L} + \frac{k_2 W h_2 (P_1 - P_2)}{\mu L} + \frac{k_3 W h_3 (P_1 - P_2)}{\mu L}$$

  - Then,

$$\bar{k}h = k_1 h_1 + k_2 h_2 + k_3 h_3$$

  - Finally,

$$\bar{k} = \sum_{i=1}^{n} \frac{k_i h_i}{h}$$



Average Permeability

- **Linear Series System**
  - The same flow rate is passing through all the layers:

$$q_1 = q_2 = q_3$$

  - The pressure difference across each layer is different, therefore:

$$P_1 - P_2 = \Delta P_1 + \Delta P_2 + \Delta P_3$$

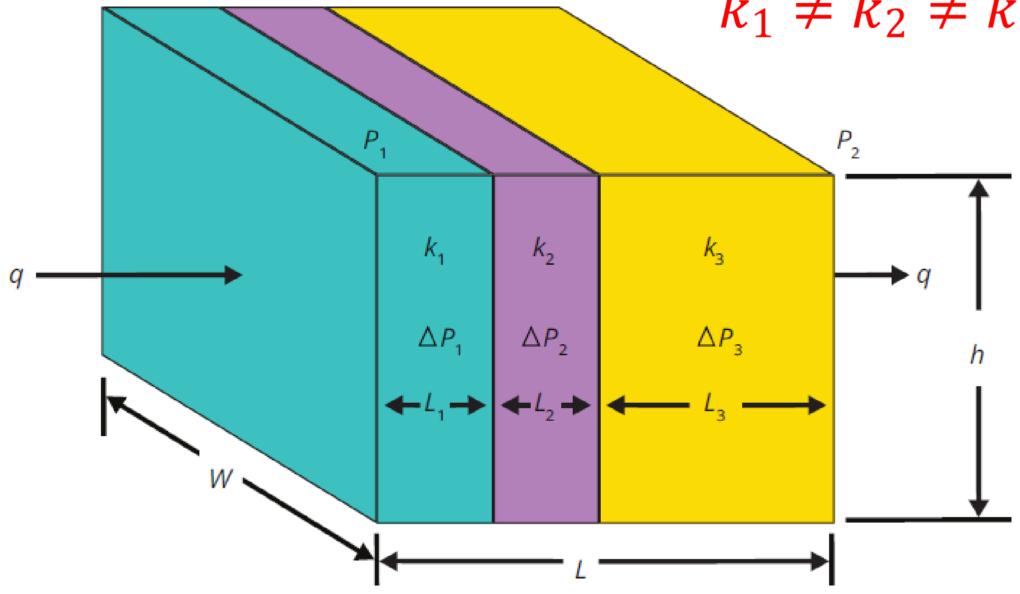
  - It is also known that:

$$L = L_1 + L_2 + L_3$$

  - We also know that the total flow rate across the system is:

$$q = \frac{\bar{k} W h (P_1 - P_2)}{\mu L}$$

$$k_1 \neq k_2 \neq k_3$$



Average Permeability

- **Linear Series System**
  - If we substitute the pressure difference from Darcy's law in:

$$P_1 - P_2 = \Delta P_1 + \Delta P_2 + \Delta P_3$$

  - It becomes:

$$P_1 - P_2 = \frac{q\mu L}{\bar{k}Wh} = \frac{q\mu L_1}{k_1 Wh} + \frac{q\mu L_2}{k_2 Wh} + \frac{q\mu L_3}{k_3 Wh}$$

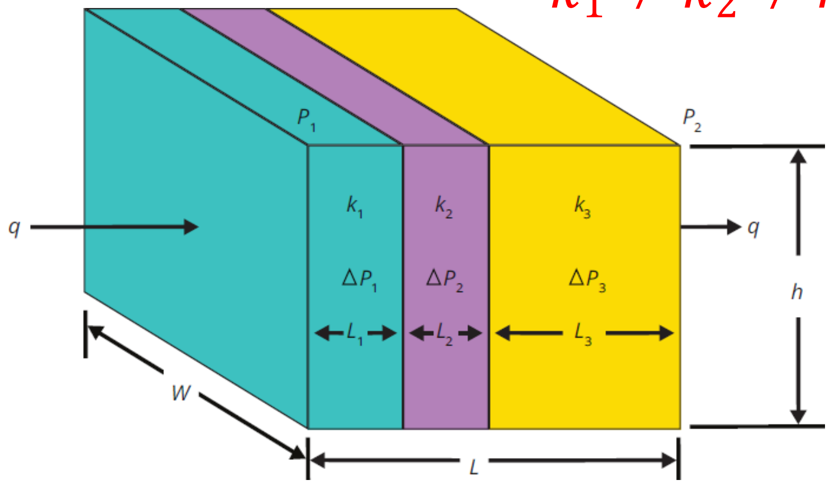
  - Then,

$$\frac{L}{\bar{k}} = \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}$$

  - Finally,

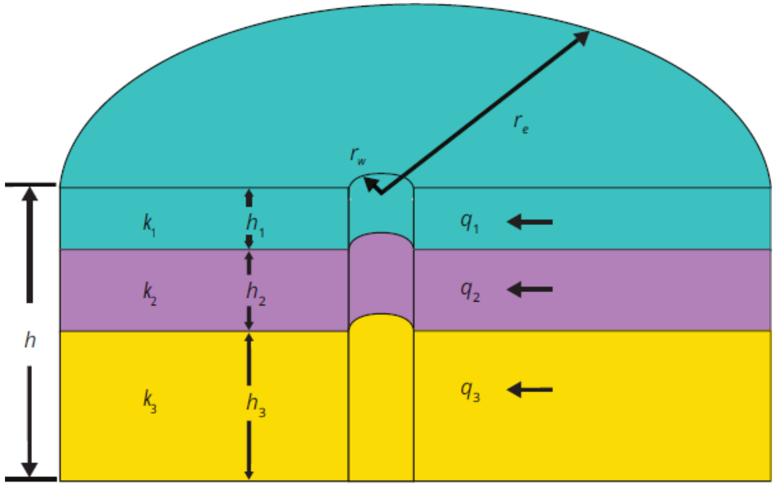
$$\bar{k} = \frac{L}{\sum_{i=1}^{n} \frac{L_i}{k_i}}$$

$k_1 \neq k_2 \neq k_3$



Average Permeability

- **Radial Parallel System**
  - The flow in such systems is similar to the flow in linear Parallel systems:



$$q = q_1 + q_2 + q_3$$

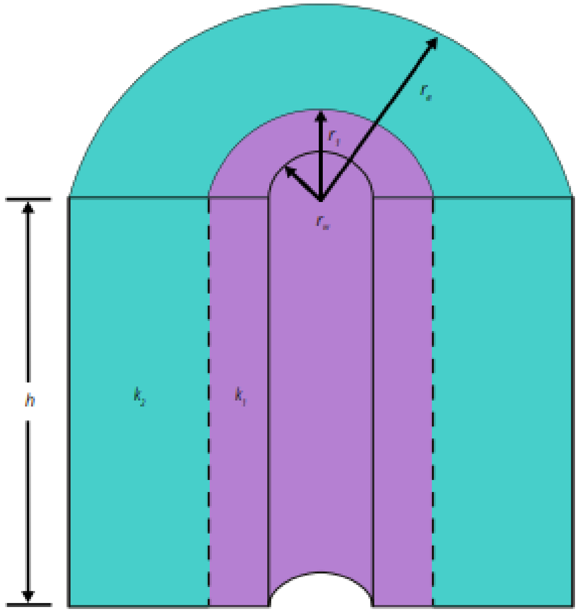
$$h = h_1 + h_2 + h_3$$

$$q = \frac{2\pi \bar{k} h \left(P_e - P_{wf}\right)}{\mu \ln\left(\frac{r_e}{r_w}\right)} = \frac{2\pi k_1 h_1 \left(P_e - P_{wf}\right)}{\mu \ln\left(\frac{r_e}{r_w}\right)} + \frac{2\pi k_2 h_2 \left(P_e - P_{wf}\right)}{\mu \ln\left(\frac{r_e}{r_w}\right)} + \frac{2\pi k_3 h_3 \left(P_e - P_{wf}\right)}{\mu \ln\left(\frac{r_e}{r_w}\right)}$$

$$\bar{k} h = k_1 h_1 + k_2 h_2 + k_3 h_3 \quad \Rightarrow \quad \bar{k} = \sum_{i=1}^{n} \frac{k_i h_i}{h}$$

Average Permeability

- **Radial Series System**
  - The flow in such systems is similar to the flow in linear series systems but with using the radial equation:



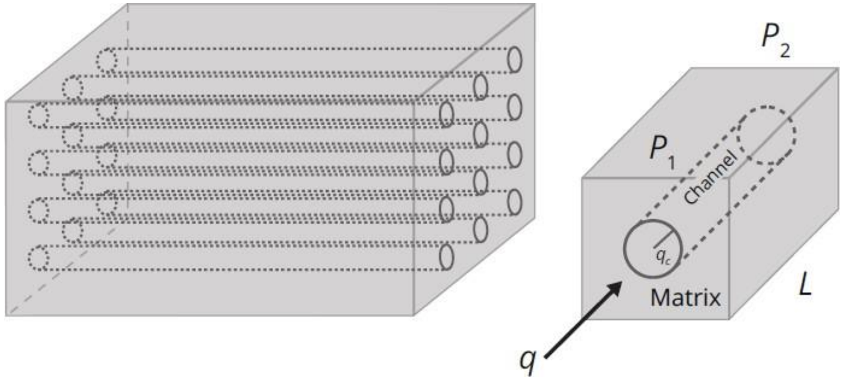
$$P_1 - P_2 = \Delta P_1 + \Delta P_2 + \Delta P_3$$

$$P_e - P_{wf} = \frac{q\mu \ ln\left(\frac{r_e}{r_w}\right)}{2\pi \overline{k} h} = \frac{q\mu \ ln\left(\frac{r_1}{r_w}\right)}{2\pi k_1 h} + \frac{q\mu \ ln\left(\frac{r_e}{r_1}\right)}{2\pi k_2 h}$$

$$\overline{k} = \frac{ln\left(\frac{r_e}{r_w}\right)}{\sigma_{i=1}^{n} \frac{ln\left(\frac{r_{(i+1)}}{r_i}\right)}{k_i}}$$

Flow in Channels and Fractures

- Channels and fractures can be superficially induced in reservoirs to increase the permeability, but some reservoirs can be naturally fractured.



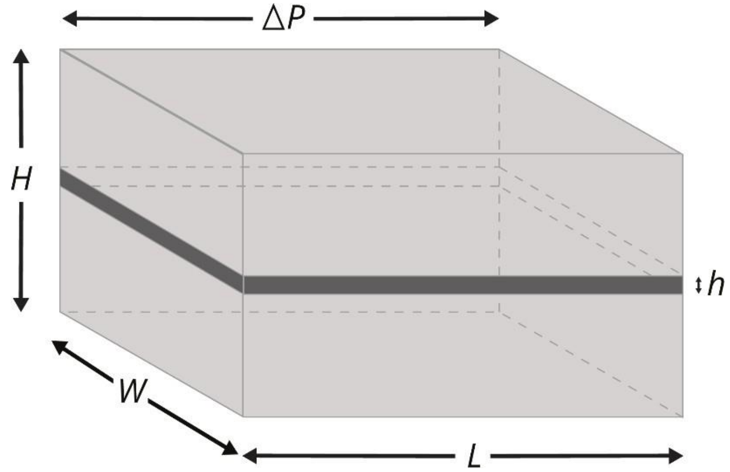
- Flow in Channels

- Channels are created in reservoirs by injecting acids to dissolve the rock in order to increase the permeability in the near-wellbore region.
- Increasing *k* will increase the flow rate of HC in the well and therefore will increase the productivity.
- Channels can have different shapes, but the simplest is the capillary tube shape, which is the wormhole effect cased by acidizing.
- The flow in this capillary tube is described by: ⟶ $q = \frac{Ar^2(P_1 - P_2)}{8\mu L}$
- If compared with Darcy's linear flow of liquids: ⟶ $\frac{kA(P_1 - P_2)}{\mu L} = q = \frac{Ar^2(P_1 - P_2)}{8\mu L}$
- Then, the permeability in channels can be found by: ⟶ $k = \frac{r^2}{8}$

Flow in Channels and Fractures

- Flow in Fractures

- Fractures can either be natural or induced, and the simplest model assumes a slab of constant thickness.



- The flow in this slab is described by: $\longrightarrow$ $$q = \frac{Ah^2(P_1 - P_2)}{12\mu L}$$

- If compared with Darcy's linear flow of liquids: $\longrightarrow$ $$\frac{kA(P_1 - P_2)}{\mu L} = q = \frac{Ah^2(P_1 - P_2)}{12\mu L}$$

- Then, the permeability in channels can be found by: $\longrightarrow$ $$k = \frac{h^2}{12}$$