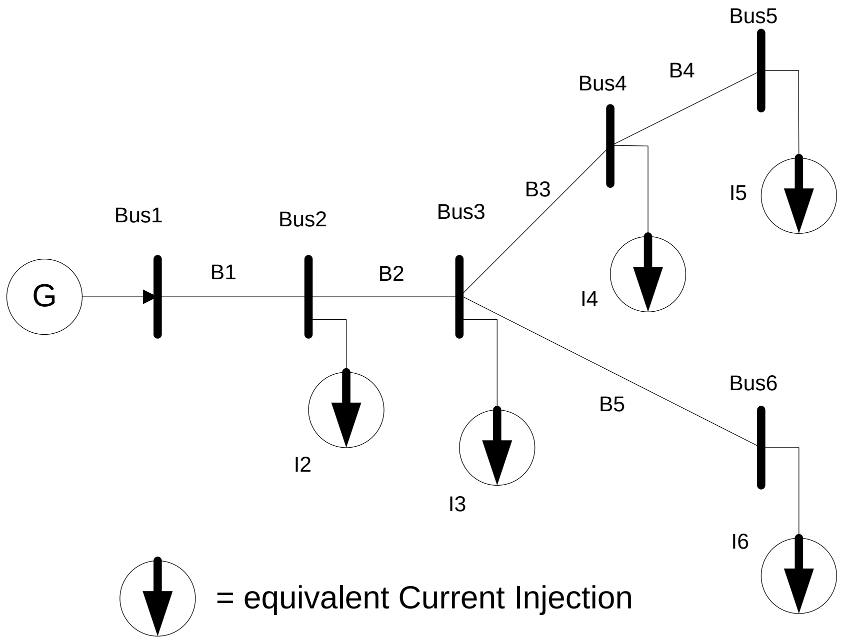
A NETWORK TOPOLOGY BASED THREE PHASE LOAD FLOW

(Balanced system)

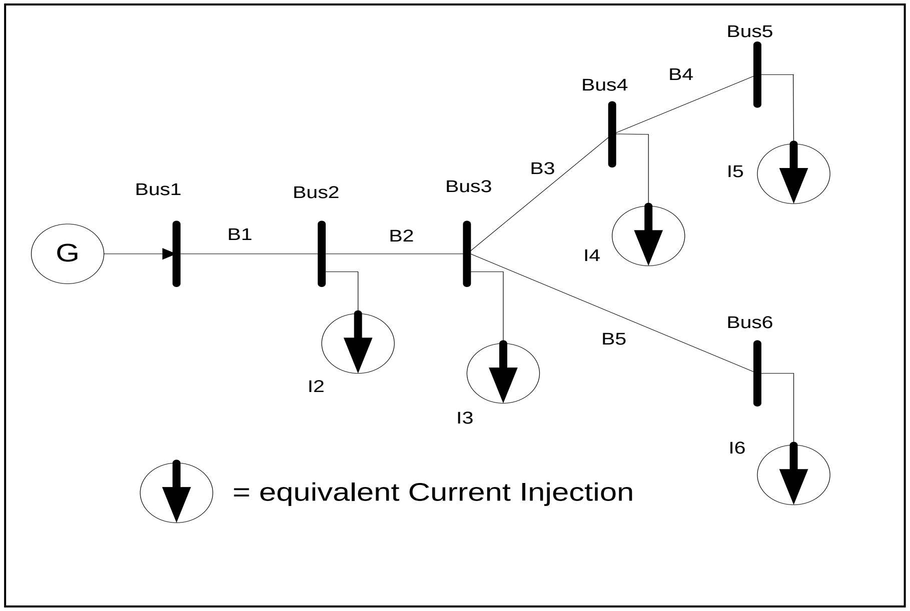


Step 1:

Build the Bus-Injection to Branch-Current (**BIBC**) matrix.

B1, B2, …. B6. Is branch current.

Look at the figure.



$$B_5 = I_6$$

$$B_4 = I_5$$

$$B_3 = I_4 + I_5$$

$$B_2 = I_4 + I_5 + I_6$$

$$B_1 = I_2 + I_3 + I_4 + I_5 + I_6$$

Furthermore, **BIBC** matrix can be obtained as

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix}$$

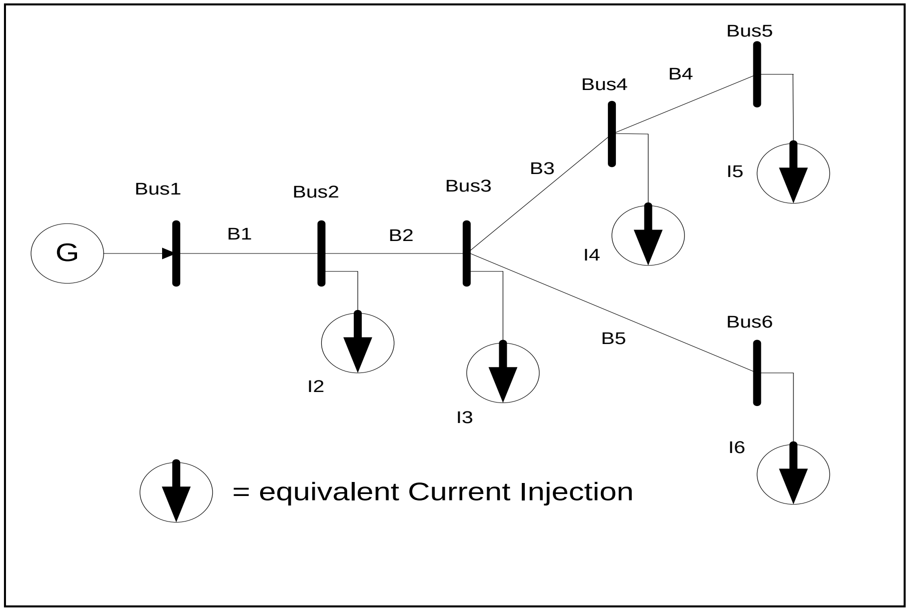
$Can\ be\ expressed\ in\ the\ general\ form\ as$

$[B] = [\boldsymbol{BIBC}][I]$ ……… (1)

Step2.

Build Branch-Current to Bus-Voltage (**BCBV**) matrix.

Look at the figure:



$$V_2 = V_1 - B_1.Z_{12}$$

$$V_3 = V_2 - B_2.Z_{23}$$

$$V_3 = V_1 - B_1.Z_{12} - B_2.Z_{23}$$

$$V_4 = V_3 - B_3.Z_{34}$$

$$V_4 = V_1 - B_1.Z_{12} - B_2.Z_{23} - B_3.Z_{34}$$

$$V_5 = V_4 - B_4.Z_{45}$$

$$V_5 = V_1 - B_1.Z_{12} - B_2.Z_{23} - B_3.Z_{34} - B_4.Z_{45}$$

$$V_6 = V_3 - B_5.Z_{26}$$

$$V_6 = V_1 - B_1.Z_{12} - B_2.Z_{23} - B_5.Z_{26}$$

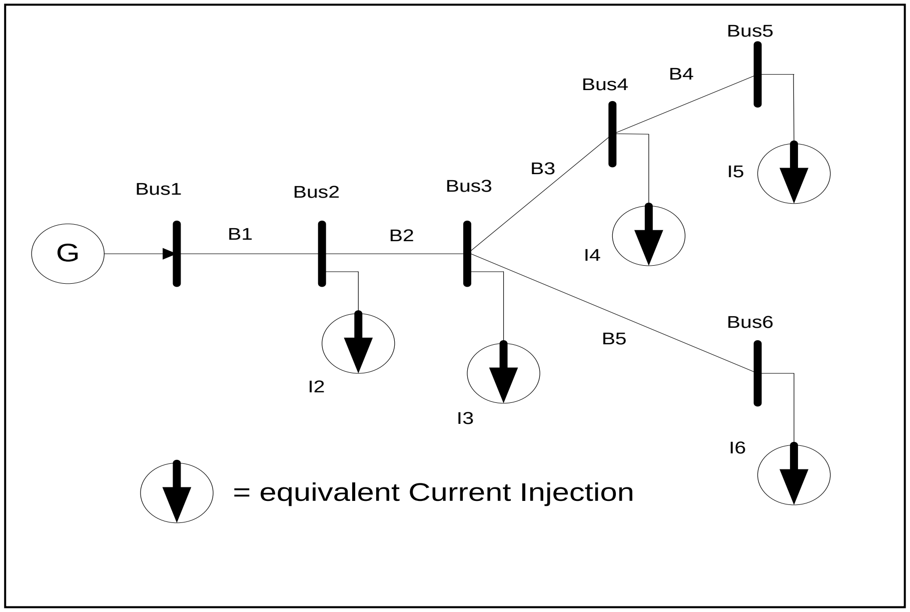
$$V_1 - V_2 = B_1.Z_{12}$$

$$V_1 - V_3 = B_1.Z_{12} + B_2.Z_{23}$$

$$V_1 - V_4 = B_1.Z_{12} + B_2.Z_{23} + B_3.Z_{34}$$

$$V_1 - V_5 = B_1.Z_{12} + B_2.Z_{23} + B_3.Z_{34} + B_4.Z_{45}$$

$$V_1 - V_6 = B_1.Z_{12} + B_2.Z_{23} + B_5.Z_{36}$$



$$\begin{bmatrix} V_1 - V_2 \\ V_1 - V_3 \\ V_1 - V_4 \\ V_1 - V_5 \\ V_1 - V_6 \end{bmatrix} = \begin{bmatrix} Z_{12} & 0 & 0 & 0 & 0 \\ Z_{12} & Z_{23} & 0 & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & Z_{45} & 0 \\ Z_{12} & Z_{23} & 0 & 0 & Z_{36} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix}$$

Rewriting in the general form, we have

$$[\Delta V] = [\boldsymbol{BCBV}][B] \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

Substituted (1) to (2), we get

$[\Delta V] = [\boldsymbol{BCBV}][\boldsymbol{BIBC}][I]$…………(3)

$[\Delta V] = [\boldsymbol{DLF}][I]$ …………(4)

Step3.

Follow the algorithm.

1. Input data.
2. Build ***BIBC*** matrix**.**
3. Build **BCBV** matrix**.**
4. Build **DLF** matrix**.**
5. Iteration k=0.
6. Iteration k=k+1.

7. Solve for three-phase power flow by using

$$I_i^k = \left(\frac{P_i + jQ_i}{V_i^k}\right)^*$$

And

$$[\Delta V^k] = [\boldsymbol{DLF}][I^k]$$

And update voltages.

$$[V^{k+1}] = [Vno\_load] - [\Delta V^k]$$

8. If $max_i\left(\left|I_i^{k+1}\right| - \left|I_i^k\right|\right) > tolerance,$ go to (6)
9. Report and end.

Reference:

1. Jen-Hao TENG, *"A Network-Topology-based Three-Phase Load Flow for Distribution Systems",* Proc. Natl.Sci.Counc. ROC(A) Vol.24, No. 4,2000.pp.259-264

ANOTHER METHOD TO BUILD BIBC MATRIX

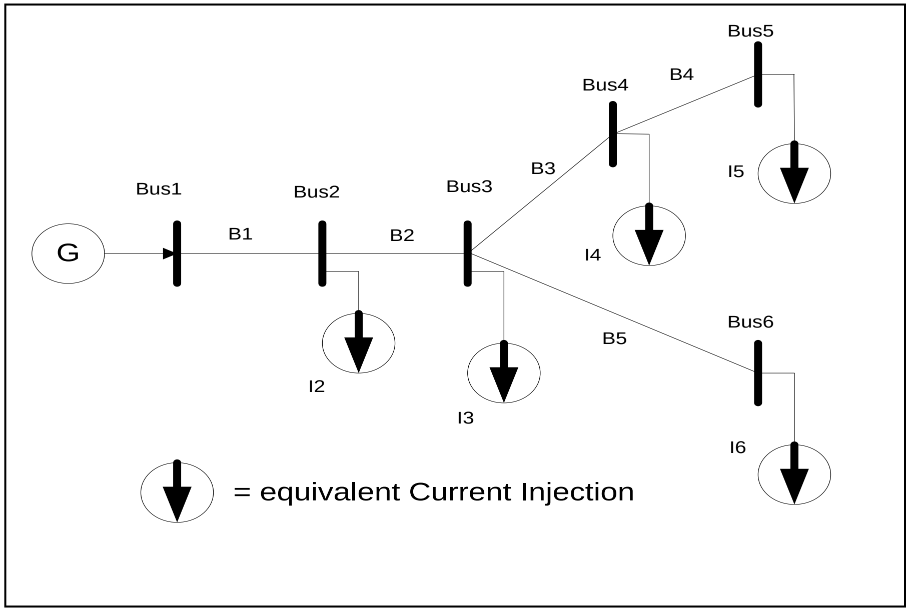
1. Branch-path incidence matrix

   The branch-path incidence matrix also called **K-Matrix**

   BIBC Matrix = - [K-Matrix]

   The elements of K-Matrix are:

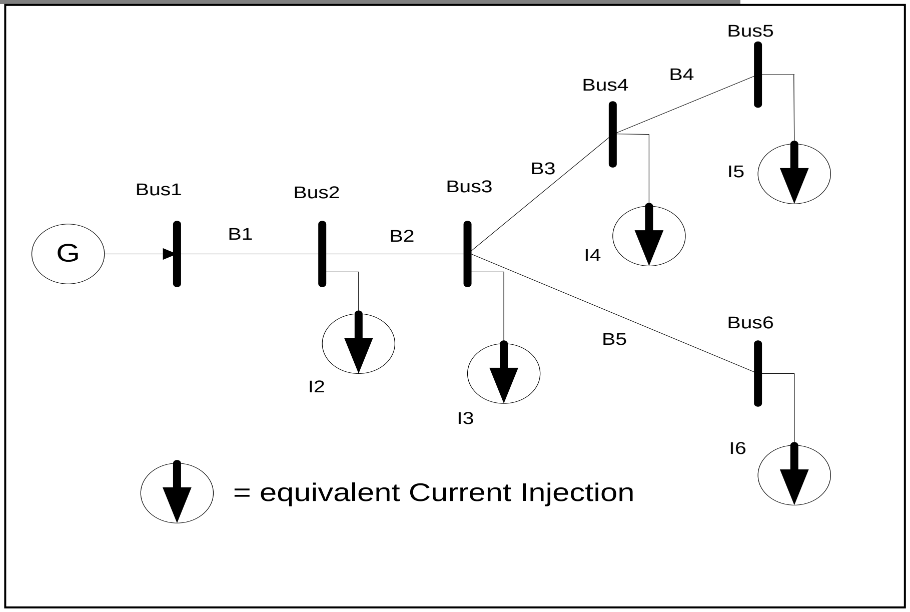
   - Row element is number of branch
   - Column element is number of buses (exclude of reference bus)



The principles of building branch-path incidence matrix (K-Matrix) is
Find the path from the bus go to the reference

K (i, j) = +1 if the branch *i* is in the path from the bus *j* to the reference node and directed in the same direction.

K (i, j) = -1 if the branch *i* is in the path from the bus *j* to the reference node and directed in the opposite direction.



$$K = \begin{array}{c} \\ B1 \\ B2 \\ B3 \\ B4 \\ B5 \end{array} \begin{array}{c} \begin{array}{ccccc} bus2 & bus3 & bus4 & bus5 & bus6 \end{array} \\ \begin{bmatrix} -1 & -1 & -1 & -1 & -1 \\ 0 & -1 & -1 & -1 & -1 \\ 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \end{array}$$

$$BIBC = -K = -\begin{bmatrix} -1 & -1 & -1 & -1 & -1 \\ 0 & -1 & -1 & -1 & -1 \\ 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

RELATIONSHIP BETWEEN *BIBC* MATRIX AND *BCBV* MATRIX

- **Look at K-Matrix**

$$\begin{matrix} & & bus2 & bus3 & bus4 & bus5 & bus6 \\ & B1 & -1 & -1 & -1 & -1 & -1 \\ & B2 & 0 & -1 & -1 & -1 & -1 \\ K = & B3 & 0 & 0 & -1 & -1 & 0 \\ & B4 & 0 & 0 & 0 & -1 & 0 \\ & B5 & 0 & 0 & 0 & 0 & -1 \end{matrix}$$

If we transpose the K-Matrix, we have

$$K' = \begin{matrix} & B1 & B2 & B3 & B4 & B5 \\ bus2 & -1 & 0 & 0 & 0 & 0 \\ bus3 & -1 & -1 & 0 & 0 & 0 \\ bus4 & -1 & -1 & -1 & 0 & 0 \\ bus5 & -1 & -1 & -1 & -1 & 0 \\ bus6 & -1 & -1 & 0 & 0 & -1 \end{matrix}$$

$$-K' = \begin{matrix} & B1 & B2 & B3 & B4 & B5 \\ bus2 & 1 & 0 & 0 & 0 & 0 \\ bus3 & 1 & 1 & 0 & 0 & 0 \\ bus4 & 1 & 1 & 1 & 0 & 0 \\ bus5 & 1 & 1 & 1 & 1 & 0 \\ bus6 & 1 & 1 & 0 & 0 & 1 \end{matrix}$$

Because

$BIBC = -K$

So

$BIBC' = -K'$

Compare $BIBC'$ with BCBV

$$BIBC' = \begin{matrix} & B1 & B2 & B3 & B4 & B5 \\ bus2 \\ bus3 \\ bus4 \\ bus5 \\ bus6 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{BCBV} = \begin{bmatrix} Z_{12} & 0 & 0 & 0 & 0 \\ Z_{12} & Z_{23} & 0 & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & Z_{45} & 0 \\ Z_{12} & Z_{23} & 0 & 0 & Z_{36} \end{bmatrix}$$

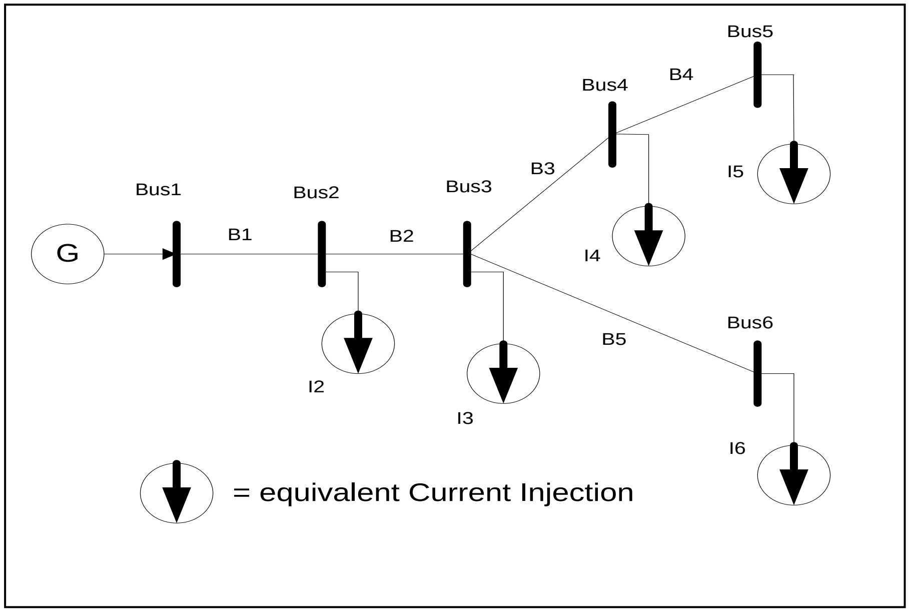
$Z_{12} = impedance\ in\ branch\ B1$

$Z_{23} = impedance\ in\ branch\ B2$

$Z_{34} = impedance\ in\ branch\ B3$

$Z_{45} = impedance\ in\ branch\ B4$

$Z_{36} = impedance\ in\ branch\ B5$



Reference:

1. Jen-Hao TENG, *“A Network-Topology-based Three-Phase Load Flow for Distribution Systems”,* Proc. Natl.Sci.Counc. ROC(A) Vol.24, No.4,2000.pp.259-264
2. T.-H. Chen, N.-C.Yang, “Three-phase power-flow by direct Zbr method for unbalanced radial distribution systems”, IET Gener.Transm.Distrib., 2009,Vol.3, Iss.10,pp.903-910.