Chapter Ten

Simulation

13.1 Overview of Simulation

- When do we prefer to develop **simulation model** over an analytic model?
  - When not all the underlying assumptions set for analytic model are valid.
  - When mathematical complexity makes it hard to provide useful results.
  - When "good" solutions (not necessarily optimal) are satisfactory.
- A simulation develops a model to numerically evaluate a system over some time period.
- By estimating characteristics of the system, *the best alternative* from *a set of alternatives under consideration* can be selected.

13.1 Overview of Simulation

- *Continuous simulation systems* monitor the system each time a change in its state takes place.
- *Discrete simulation systems* monitor changes in a state of a system at discrete points in time.
- Simulation of most practical problems requires the use of a computer program.

13.1 Overview of Simulation

- Approaches to developing a simulation model
  - Using add-ins to Excel such as @Risk or Crystal Ball
  - Using general purpose programming languages such as: FORTRAN, PL/1, Pascal, Basic.
  - Using simulation languages such as GPSS, SIMAN, SLAM.
  - Using a simulator software program.
- Modeling and programming skills, as well as knowledge of statistics are required when implementing the simulation approach.

10.2 Monte Carlo Simulation

- Monte Carlo simulation generates random events.
- Random events in a simulation model are needed when the input data includes random variables.
- To reflect the relative frequencies of the random variables, the *random number mapping* method is used.

JEWEL VENDING COMPANY –

an example for the random mapping technique

- Jewel Vending Company (JVC) installs and stocks vending machines.

- Bill, the owner of JVC, considers the installation of a certain product ("Super Sucker" jaw breaker) in a vending machine located at a new supermarket.

JEWEL VENDING COMPANY –
an example of the random mapping technique

- Data
  - The vending machine holds 80 units of the product.
  - The machine should be filled when it becomes half empty.
  - Daily demand distribution is estimated from similar vending machine placements.
    - P(Daily demand = 0 jaw breakers) = 0.10
    - P(Daily demand = 1 jaw breakers) = 0.15
    - P(Daily demand = 2 jaw breakers) = 0.20
    - P(Daily demand = 3 jaw breakers) = 0.30
    - P(Daily demand = 4 jaw breakers) = 0.20
    - P(Daily demand = 5 jaw breakers) = 0.05

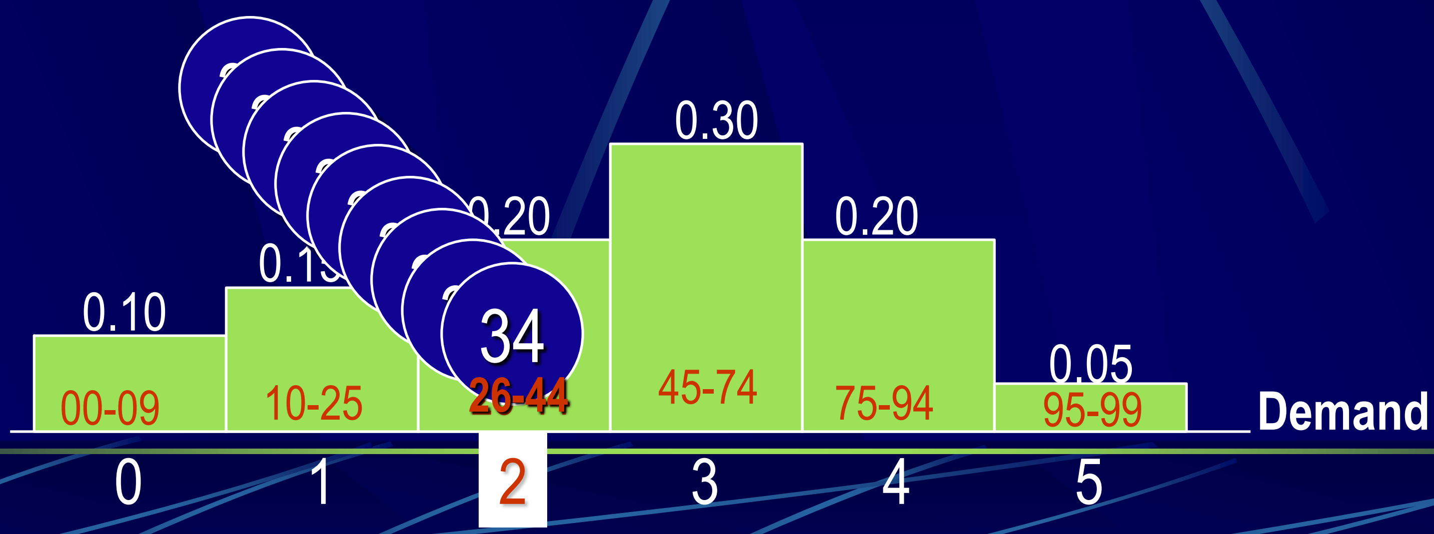
Bill would like to estimate the expected number of days it takes for a filled machine to become half empty.

Random number mapping – The Probability function Approach

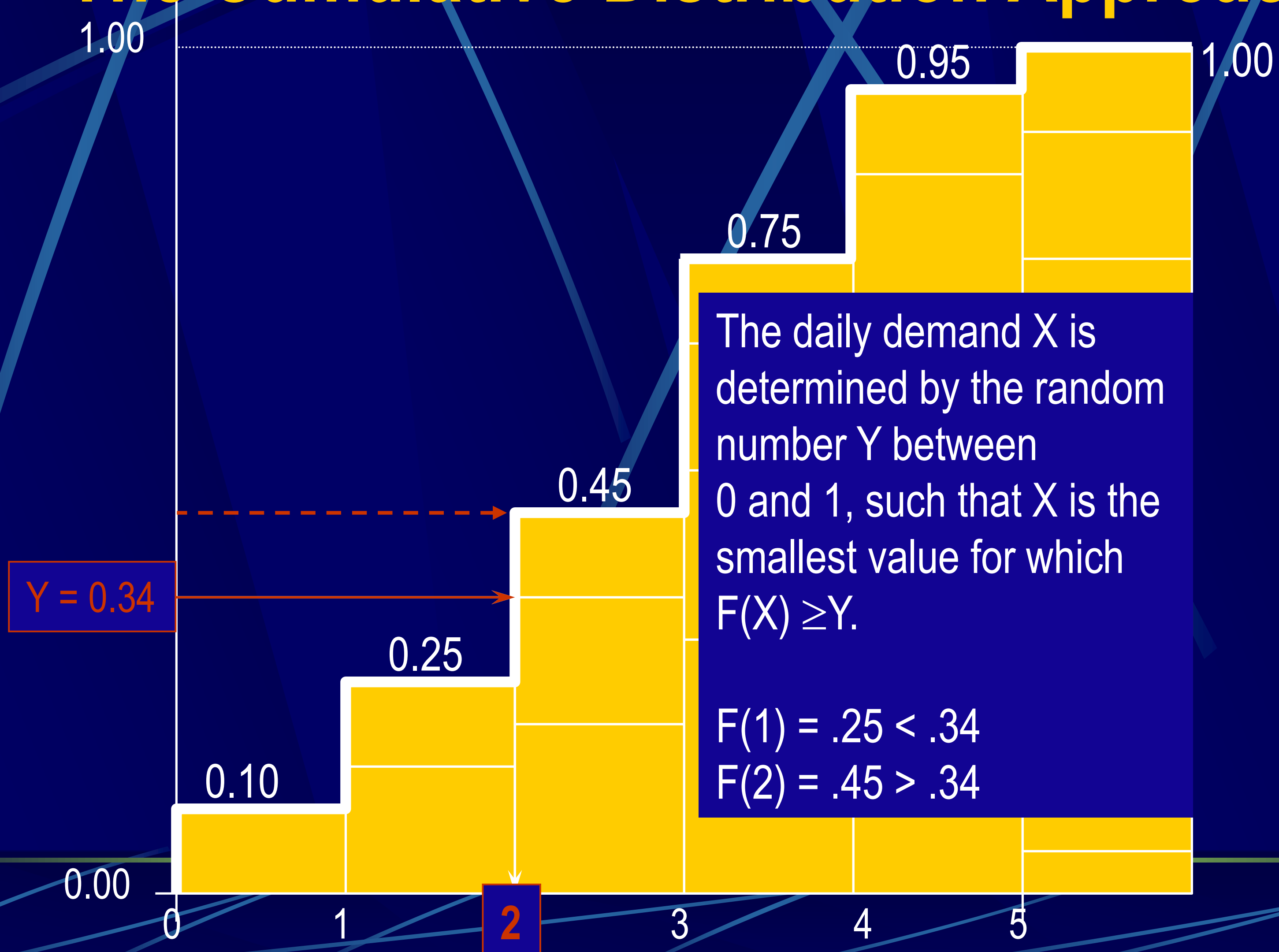
Random number mapping uses the probability function to generate random demand.

A number between 00 and 99 is selected randomly.

The daily demand is determined by the mapping demonstrated below.



Random number mapping – The Cumulative Distribution Approach



The daily demand X is determined by the random number Y between 0 and 1, such that X is the smallest value for which $F(X) \geq Y$.

$F(1) = .25 < .34$
$F(2) = .45 > .34$

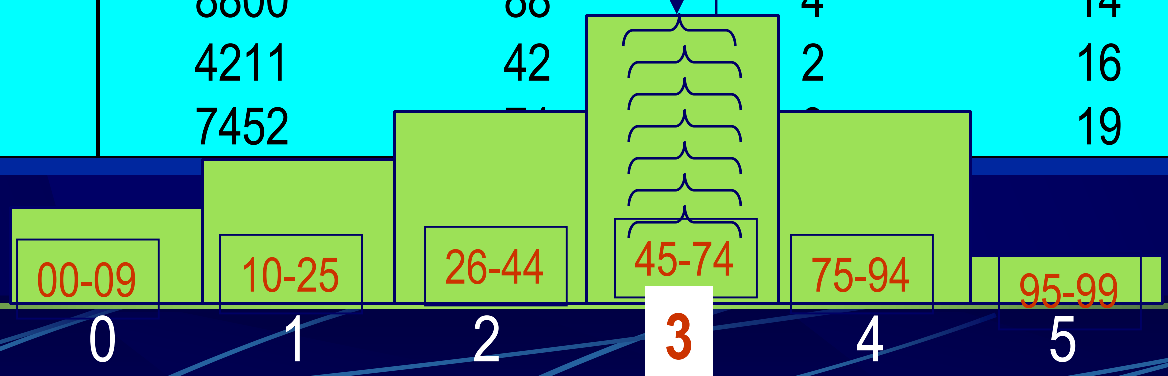
Simulation of the JVC Problem

- A random demand can be generated by hand (for small problems) from a table of pseudo random numbers.
- Using Excel a random number can be generated by
  - The RAND() function
  - The random number generation option (Tools>Data Analysis)

Simulation of the JVC Problem

- An illustration of generating a daily random demand.
- Since we have two digit probabilities, we use the first two digits of each random number.

Day	Random Number	Two First Digits	Demand	Total Demand to Date
1	6506	65	3	3
2	7761	77	4	7
3	6170	61	3	10
4	8800	88	4	14
5	4211	42	2	16
6	7452	[illegible]	[illegible]	19



Simulation of the JVC Problem

The simulation is repeated and stops once total demand reaches 40 or more.

Day	Random Number	Two First Digits	Demand	Total Demand to Date
1	6506	65	3	3
2	[illegible]	[illegible]	[illegible]	7
3	[illegible]	[illegible]	[illegible]	10
4	[illegible]	[illegible]	[illegible]	14
5	[illegible]	[illegible]	[illegible]	16
6	7452	74	3	19

The number of "simulated" days required for the total demand to reach 40 or more is recorded.

Simulation Results and Hypothesis Tests

- The purpose of performing the simulation runs is to find the average number of days required to sell 40 jaw breakers.
- Each simulation run ends up with (possibly) a different number of days.
- A hypothesis test is conducted to test whether or not μ = 16.

Null hypothesis	$H_0 : \mu = 16$
Alternative hypothesis	$H_A : \mu \neq 16$

Simulation Results and Hypothesis Tests

- The test:
  - Define α (the significance level).
  - Let n be the number of replication runs.
  - Build the t-statistic →

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

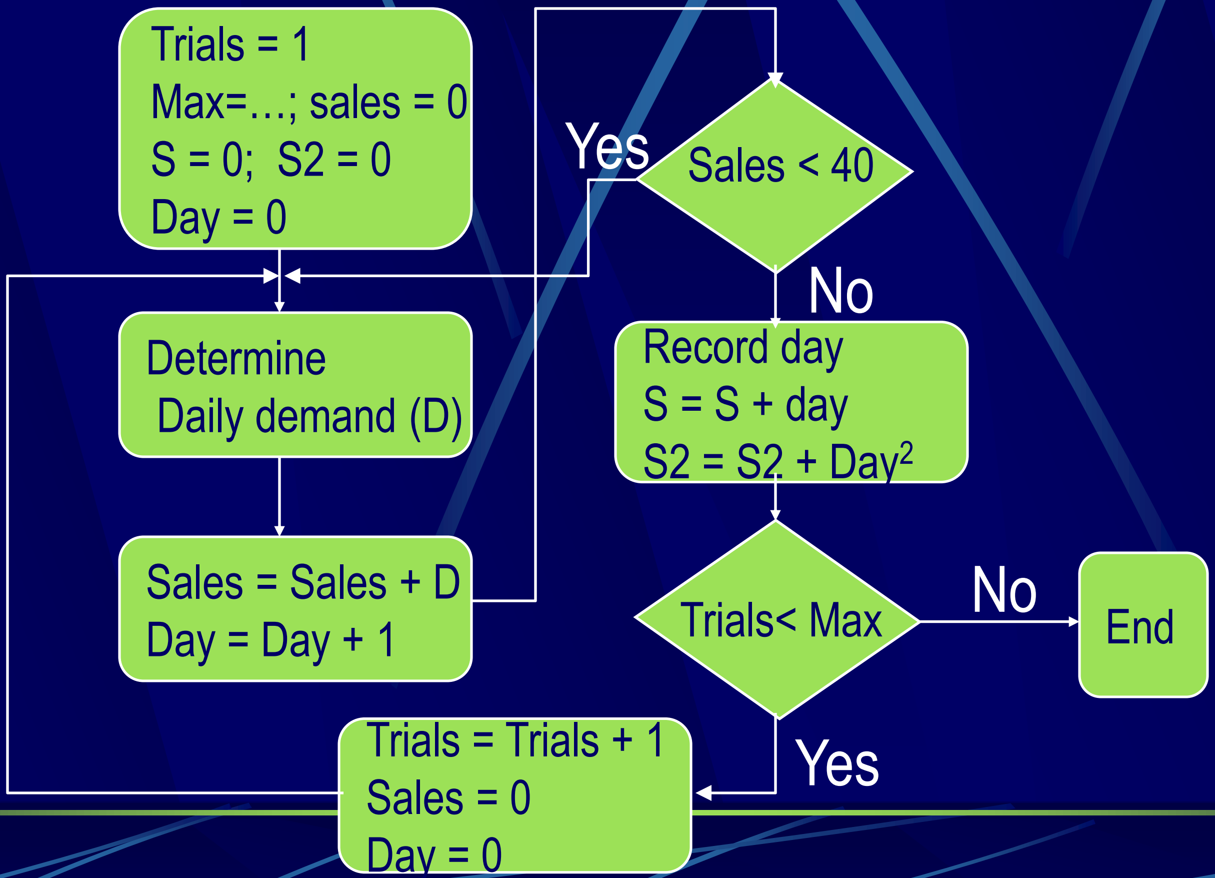
The t-statistic can be used if the random variable observed (number of day required for the total demand to be 40 or more) is normally distributed, while the standard deviation is unknown.

  - Reject H_0 if $t > t_{\alpha/2}$ or $t < -t_{\alpha/2}$

    ($t_{\alpha/2}$ has n-1 degrees of freedom.)

JVC – A Flow Chart

Flow charts help guide the simulation program



JVC – Excel Spreadsheet

Number of Days to Sell 40 Jaw Breakers = 19

=MAX(A5:A105
)

Day	Demand	Cumulative Demand
1	2	2
2	2	4
3	3	7
4	2	9
5	3	12
6	0	12
7	3	15
8	0	[illegible]
9	2	[illegible]
10	2	19
11	4	23
12	1	24
13	4	28
14	2	30
15	3	33

=IF(C5<40,A5+1,"")

=IF(C5<40,B6+C5,""
)

=IF(C5<40,VLOOKUP(RAND(),$K$6:$L$11,2),"")

K	L
0	0
0.1	1
0.25	2
0.45	3
0.75	4
0.95	5

VLOOKUP TABLE Enter this data

Drag A5:C5 to A105:C105

JVC – Excel Spreadsheet

	A	B	C	D	E	F	G	H
1	**Replication**	**Days**		*Days*				
2	1	18						
3	2	15		Mean	16.6			
4	3	14		Standard Error	0.541603			
5	4	18		Median	17		t	1.107823
6	5	17		Mode	18		p-value	0.296665
7	6	17		Standard Deviation	1.712698			
8	[illegible]	[illegible]		[illegible]	[illegible]			
9	[illegible]	[illegible]		[illegible]	[illegible]			
10	[illegible]	[illegible]		[illegible]	[illegible]			
11	[illegible]	[illegible]		[illegible]	[illegible]			
12				[illegible]	14			
13				[illegible]	19			
14				[illegible]	166			
15				Count	10			

=(E3-16)/E4

=TDIST(ABS(H5),9,2)

- The p-value =.2966… This value is quite high compared to any reasonable significance level.
- **Based on this data there is insufficient evidence to infer that the mean number of days differs from 16.**

10.3 Simulation Modeling of Inventory Systems

- Inventory simulation models are used when underlying assumptions needed for analytical solutions are not met.
- Typical inputs into the simulation model are
  - Order cost
  - Holding cost
  - Lead time
  - Demand distribution

10.3 Simulation Modeling of Inventory Systems – continued

- Frequently, the *Fixed-Time Simulation* approach is appropriate for the modeling of inventory problems.
  - The system is monitored periodically.
  - The activities associated with demand, orders, and shipments are determined, and the system is updated accordingly.
- Typical output is the average total cost for a given inventory policy.

ALLEN APPLIANCE COMPANY – An example of an inventory simulation

- Allen Appliance stocks and sells the KitchenChef electric mixer.
- Allen wishes to determine an optimal inventory policy for the mixer based on the following data:
- Data
  - Unit cost is $200, and selling price is $260.
  - Annual holding cost rate is 26%.
  - Orders are placed at the end of a week, and arrive at the beginning of a week, two weeks later.
  - Ordering cost is $45 per order.
  - Backorder cost is $5 per unit per week.
  - Backorder administrative cost is $2 per unit.

Continued…

ALLEN APPLIANCE COMPANY

- Distribution of…
  - The number of customers who arrive weekly:

    P(Arrivals = 0) = .10
    P(Arrivals = 1) = .30
    P(Arrivals = 2) = .25
    P(Arrivals = 3) = .20
    P(Arrivals = 4) = .15

  - The demand per customer:

    P(Demand = 1) = 0.10
    P(Demand = 2) = 0.15
    P(Demand = 3) = 0.40
    P(Demand = 4) = 0.35

AAC – The Planned Shortage Model

- Let us assume first a constant demand rate, and use the planned shortage model. We need to calculate the following parameters:
  - Average weekly demand =
    (Average number of customers/week)(Average demand/customer) =
    [.10(0)+.25(2)+.2(3)+.15(4)][.10(1)+.15(2)+.40(3)+.35(4)] =
    (2)(3) = 6
  - Holding cost per unit per week =
    (Ann. Holding cost rate)(Unit cost)/52 = .26(200)/52 = $1

AAC – The Planned Shortage Model

- Using the template inventory.xls for the planned shortage model, and assuming a constant demand of 6 units per week (312 per year) we have:
  - Optimal ordering policy:
    - Q^* = 24.88 (rounded to 25)
    - S^* = 2.15 (rounded to to 2); Reorder when inventory is at a level of 10.
  - Total annual cost $TC(Q^*,S^*)$ = $63582.48

AAC – The Simulation Model

- Because demand is uncertain, a simulation models has been developed.
- A continuous review (R,Q) system is studied first, where R = 10 and Q = 25.

AAC – The Simulation Model

- The random number mapping associated with the distributions are:

Number of Arrivals	Probability	Random # mapping
0	.10	00 – 09
1	.30	10 – 39
2	.25	40 – 64
3	.20	65 – 84
4	.15	85 – 99

Demand/customer	Probability	Random # mapping
1	.10	00 – 09
2	.15	10 – 24
3	.40	25 – 64
4	.35	65 – 99

AAC – The Simulation Logic

- The simulation keeps track of the following quantities:
  - Beginning inventory for the week = Ending inventory of the previous week + order received.
  - Number of retailers arriving, their demand, and the total weekly demand.
  - Ending inventory for the week = Beginning inventory + order received – weekly demand.

AAC – The Simulation Logic

- The simulation determines whether or not an order should be placed as follows:
  - Is the ending inventory$\leq$10 and is there no outstanding order? If so, place an order and keep track of the lead time.
- The simulation calculates the Weekly cost:
  - Ordering cost (if applicable) + Holding cost (if ending inventory > 0) + Backorder cost (if ending inventory < 0).

AAC – 10 week simulation results

Week	Beginning Inventory	Random # (Col. 1)	# of Customers	Random # (Col. 2)	Customer Demand	Weekly Demand	Ending Inventory	Order Placed?	Lead Time	Weekly Cost
1	25	65	3	33	3					
				98	4					
				26	3	10	15	No	–	$15
2	15	77	3	91	4					
				96	4					
				48	3	11	4	Yes	2	$49
3	4	61	2	82	4					
				27	3	7	-3	No	1	$21

- Initial inventory = 25.
- Weekly cost = Order cost (if any) + 1(Stock on hand) + 2(New back orders) + 5(Total backorders)
- Total cost for 10 weeks = $415 (weekly average = $41.5).

AAC – 1000 weeks of simulation spreadsheet results

INPUTS					
Q =	25	Ch =	1		
R =	10	Co =	45		
		Cs =	5		
		Cb =	2		
OUTPUT					
Average Cost =	33.109				

Day	Start of Week Inventory	# of Customer Arrivals	Total Demand	End of Week Inventory	Total Cost
1	25	3	9	16	16
2	16	2	7	9	54
3	9	3	11	-2	14
4	-2	1	4	-6	38
5	19	0	0	19	19
6	19	1	4	15	15
7	15	1	2	13	13

10.4 Simulation of a Queuing System

- In queuing systems time itself is a random variable. Therefore, we use the *next event simulation* approach.
- The simulated data are updated each time a new event takes place (not at a fixed time periods.)
- The *process interactive approach* is used in this kind of simulation (all relevant processes related to an item as it moves through the system, are traced and recorded).

CAPITAL BANK
An example of queuing system simulation

- Capital Bank is considering opening the bank on Saturdays morning from 9:00 a.m.
- Management would like to determine the waiting time on Saturday morning based on the following data:

CAPITAL BANK

- Data:
  - There are 5 teller positions of which only three will be staffed.
    - Ann Doss is the head teller, experienced, and fast.
    - Bill Lee and Carla Dominguez are associate tellers less experienced and slower.

CAPITAL BANK

- Data:
  - Service time distributions:

Ann's Service Time Distribution

Service Time	Probability
.5 minutes	.05
1	.10
1.5	.20
2	.30
2.5	.20
3	.10
3.5	.05

Bill and Carla's Service Time Distribution

Service Time	Probability
1 minute	.05
1.5	.15
2	.20
2.5	.30
3	.10
3.5	.10
4	.05
4.5	.05

CAPITAL BANK

- Data:
  - Customer inter-arrival time distribution

inter-arrival time	Probability
.5 Minutes	.65
1	.15
1.5	.15
2	.05

  - Service priority rule is first come first served
- A simulation model is required to analyze the service .

CAPITAL BANK – Solution

- Calculating expected values:
  - E(inter-arrival time) = .5(.65)+1(.15+1.5(.15)+2(.05) = .80 minutes [75 customers arrive per hour on the average, (60/.8=75)]
  - E(service time for Ann) = .1(.05)+1(.10)+…+3.5(.05) = 2 minutes [Ann can serve 60/2=30 customers per hour on the average]
  - E(Service time for Bill and Carla) = 1(.05)+1.5(.15)+…+4.5(.05) = 2.5 minutes [Bill and Carla can serve 60/2.5=24 customers per hour on the average].

CAPITAL BANK – Solution

- To reach a steady state the bank needs to employ all the three tellers (30+2(24) = 78 > 75).

Customer Inter-arrival Time

Time	Random #'s
.5 minutes	00 – 64
1 minute	65 – 79
1.5 minutes	80 – 94
2 minutes	95 – 99

Ann's Service Time

Time	Random #'s
.5 minutes	00 – 04
1 minute	05 – 14
1.5 minutes	15 – 34
2 minutes	35 –64
2.5 minutes	65 – 84
3 minutes	85 – 94
3.5 minutes	95 - 99

Bill and Carla's Service Time

Time	Random #'s
1 minute	00 – 04
1.5 minutes	05 – 19
2 minutes	20 – 39
2.5 minutes	40 – 69
3 minutes	70 - 79
3.5 minutes	80 – 89
4 minutes	90 – 94
4.5 minutes	95 – 99

CAPITAL BANK – The Simulation logic

- If no customer waits in line, an arriving customer seeks service by a free teller in the following order: Ann, Bill, Carla.
- If all the tellers are busy the customer waits in line and takes then the next available teller.
- The waiting time is the time a customer spends in line, and is calculated by

[Time service begins] minus *[Arrival Time]*

CAPITAL – Simulation Demonstration

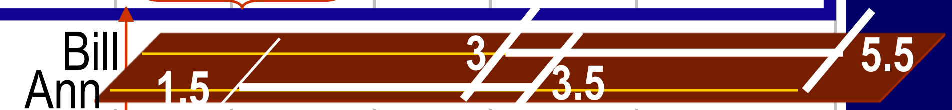
Customer	Random Number	Arrival Time	Random Number	Ann Start	Ann Finish	Bill Start	Bill Finish	Carla Start	Carla Finish	Waiting Time
1	89	1.5	63	1.5	3.5					0
2	88	3	46			3	5.5			0
7	26	7	59	7.5	9.5					0.5
8	16	7.5	28			8.5	10.5			1
9	40	8	79					9	12	1
10	65	9	64	9.5	11.5					0.5
11	61	9.5	33			10.5	12.5			1

Mapping Interarrival time
80 – 94 → 1.5 minutes

Mapping Ann's Service time
35 – 64 → 2 minutes

CAPITAL – Simulation Demonstration

Customer	Random Number	Arrival Time	Random Number	Ann Start	Ann Finish	Bill Start	Bill Finish	Carla Start	Carla Finish	Waiting Time
1	89	1.5	63	1.5	3.5					0
2	88	3	46			3	5.5			0
7	26	7	59	7.5	9.5					0.5
8	16	7.5	28			8.5	10.5			1
9	40	8	79					9	12	1
10	65	9	64	9.5	11.5					0.5
11	61	9.5	33			10.5	12.5			1



Mapping Interarrival time
80 – 94 → 1.5 minutes

Mapping Bill's Service time
40 – 69 → 2.5 minutes

CAPITAL – Simulation Demonstration

	Random	Arrival	Random	Ann		Bill		Carla		Waiting
Customer	Number	Time	Number	Start	Finish	Start	Finish	Start	Finish	Time
1	89	1.5	63	1.5	3.5					0
2	88	3	46			3	5.5			0
								[Waiting time →]		
7	26	7	59	7.5	9.5					0.5
8	16	7.5	28			8.5	10.5			1
9	40	8	79					9	12	1
10	65	9	64	9.5	11.5					0.5
11	61	9.5	33			10.5	12.5			1

CAPITAL – 1000 Customer Simulation

Average Waiting Time in Line =				1.670							
Average Waiting Time in System =				3.993							
				Ann		**Bill**		**Carla**		**Waiting Time Line**	**Waiting Time System**
Customer	**Random Number**	**Arrival Time**	**Random Number**	**Start**	**Finish**	**Start**	**Finish**	**Start**	**Finish**		
1	0.87	1.5	0.96	1.5	5					0	3.5
2	0.18	2.0	0.76			2	5			0	3.0
3	0.49	2.5	0.78					2.5	5.5	0	3.0
4	0.86	4.0	0.49	5	7					1	3.0
5	0.54	4.5	0.85			5	8.5			0.5	4.0
6	0.61	5.0	0.55					5.5	8	0.5	3.0
7	0.91	6.5	0.90	7	10					0.5	3.5
8	0.64	7.0	0.62					8	10.5	1	3.5

CAPITAL – 1000 Customer Simulation

Average Waiting Time in Line =				1.670					
Average Waiting Time in System =				3.993					
				Ann		Bill	Carla	Waiting	Waiting
	Random	Arrival	Random					Time	Time

- This simulation estimates two performance measures:
  - Average waiting time in line (W_q) = 1.67 minutes
  - Average waiting time in the system W = 3.993 minutes
- To determine the other performance measures, we can use Little's formulas:
  - Average number of customers in line L_q =(1/.80)(1.67) = 2.0875 customers
  - Average number of customers in the system = (1/.80)(3.993) = 4.99 customers.

Average inter-arrival time = .80 minutes.

Mapping for Continuous Random Variables

- Example
  - The Explicit inverse distribution method can be used to generate a random number X from the exponential distribution with $\mu = 2$ (i.e. service time is exponentially distributed, with an average of 2 customers per minute).
    - Randomly select a number from the uniform distribution between 0 and 1. The number selected is Y = .3338.
    - Solve the equation: $X = F^{-1}(Y) = -(1/\mu)\ln(1 - Y) = -(1/2)\ln(1-.3338) = .203$ minutes.

Mapping for Continuous Random Variables – Using Excel

	A	B	C
1	Mean =	2	
2			
3	**Replication**	**Random Number**	**Service Time**
4	1	0.3874	0.2450
5	2	0.0549	0.0282
6	3	0.5173	0.3642
7	4	0.5491	0.3983
8	5	0.9826	2.0245
9	6	[illegible]	1.4073
10	7	[illegible]	[illegible]
11	8	[illegible]	[illegible]
12	9	[illegible]	[illegible]
13	10	0.7735	[illegible]

=RAND()
Drag to cell B13

=-LN(1-B4)/$B$1
Drag to cell C13

Random numbers and Excel

- Excel can generate continuously distributed random numbers for various distribution.
  - Normal =NORMINV
  - Beta: =BETAINV
  - Chi squared: =CHIINV
  - Gamma: =GAMMAINV

Random numbers
Normally distributed by Excel –

	A	B
1	Mean =	35
2	Standard Deviation =	3
3		
4	**Car**	**Speed**
5	1	36.94
6	2	32.68
7	3	32.40
8	4	31.88
9	5	34.72
10	6	35.02
11	7	35.27
12	8	39.16
13	9	31.58
14	10	40.81
15	11	33.10
16	12	35.17
17	13	36.63
18	14	37.39

=NORMINV(RAND(),$B$1,$B$2)
Drag to cell B24

Simulation of an M / M / 1 Queue

- Applying the *process interaction approach* we have:
  - New arrival time = Previous arrival time + Random interarrival time.
  - Service finish time = Service start time + Random service time.
  - A customer joins the line if there is a service in progress (its arrival time < current service finish time).
  - A customer gets served when the server becomes idle.
  - Waiting times and number of customers in line and in the system are continuously recorded.

LANFORD SUB SHOP
An example of the M/M/1 queuing simulation

- Lanford Sub Shop sells sandwiches prepared by its only employee, the owner Frank Lanford.
- Frank can serve a customer in 1 minute on the average according to an exponential distribution.
- During lunch time, 11:30 a.m. to 1:30 p.m., an average of 30 customers an hour arrive at the shop according to a Poisson distribution.

Using simulation, Frank wants to determine the average time a customer must wait for service

LANFORD SUB SHOP - Solution

- Input Data

  $\lambda = 30, \ \mu = 60.$

- Data generated by the simulation:
  - C# = The number of the arriving customer.
  - R#1 = The random number used to determine interarrivals.
  - IAT = The interarrival time.
  - AT = The arrival time for the customer.
  - TSB = The time at which service begins for the customer.
  - WT = The waiting time a customer spends in line.
  - R#2 = The random number used to determine the service time.
  - ST = The service time.
  - TSE = The time at which service end for the customer

LANFORD SUB SHOP – Simulation for first 10 Customers

Arrival Time · Time Service Begins · Service Time · Time Service Ends

C#	R#1	IAT	AT	TSB	WT	R#2	ST	TSE
1	0.6506	2.10	2.10	2.10	0	0.7761	1.5	3.6
2	0.6170	1.92	4.02	4.02	0	0.8800	2.12	6.14
3	0.4211	1.09	5.11	6.14	1.03	0.7452	1.37	7.51
4	0.1182	0.25	5.36	7.51	2.15	0.4012	0.51	8.02
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.59	0.6299	0.99	9.01
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.99	0.1085	0.11	9.12
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.73	0.6969	1.19	10.31
8	0.1696	0.37	7.76	10.31	2.55	0.0267	0.03	10.34
9	0.3175	0.76	8.52	10.34	1.82	0.7959	1.59	11.93
10	0.4958	1.37	9.89	11.93	2.04	0.4281	0.56	12.49

Average waiting time =
(0 + 0 +1.03 + ... +2.04) / 10 = 1.59

The interarrival time = - ln(1-0.4211) / 30 = 0.0182 hours = 1.09 minutes

The explicit inverse method

End of service = 6.14+1.37

Waiting time = 6.14 - 5.11

Arrival time of customer 3 = Arrival time of customer 2 + 1.09 = 4.02 + 1.09

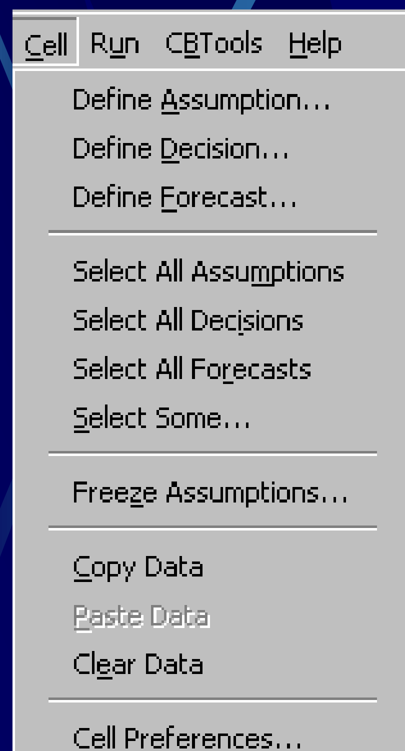
LANFORD SUB SHOP – Simulation for first 1000 Customers

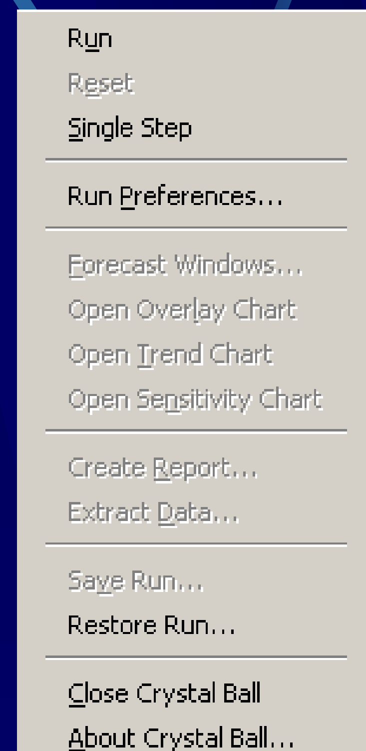
	A	B	C	D	E	F	G
1	Arrival Rate (lambda) per hour =			30			
2	Service Rate (mu) per hour =			60			
3							
4	Average Waiting Time (in minutes) =				0.970982		
5							
6	Cust #	IAT	AT	TSB	WT	ST	TSE
7							
8	1	3.185907	3.185907	3.185907	0	0.161422	3.34733
9	2	0.907386	4.093294	4.093294	0	1.169643	5.262937
10	3	0.448395	4.541689	5.262937	0.721248	0.191208	5.454144
11	4	0.01861	4.560299	5.454144	0.893845	0.860468	6.314613
12	5	1.188251	5.748551	6.314613			
13	6	0.66342	6.411971	7.601137			
14	7	0.906348	7.318318	9.994628			
15	8	1.205453	8.523771	10.00124			
16	9	4.724972	13.24874	13.24874			
17	10	1.362563	14.61131	14.61131			
18	11	1.208411	15.81972	17.64179			
19	12	5.357083	21.1768	21.1768			
20	13	0.79991	21.97671	22.38254			

Cell	Value	Formula
E4	Wq	=AVERAGE(E8:E1007)
Row 8		
A8	Customer Number	=A7+1
B8	Customer Inter-arrival Time	=-LN(1-RAND())/($D$1/60)
C8	Arrival Time	=B8+C7
D8	Time Service Begins	=MAX(C8,G7)
E8	Waiting Time	=D8-C8
F8	Service Time	=-LN(1-RAND())/($D$2/60)
G8	Time Service Ends	=D8+F8

Conducting Simulation using Crystal Ball

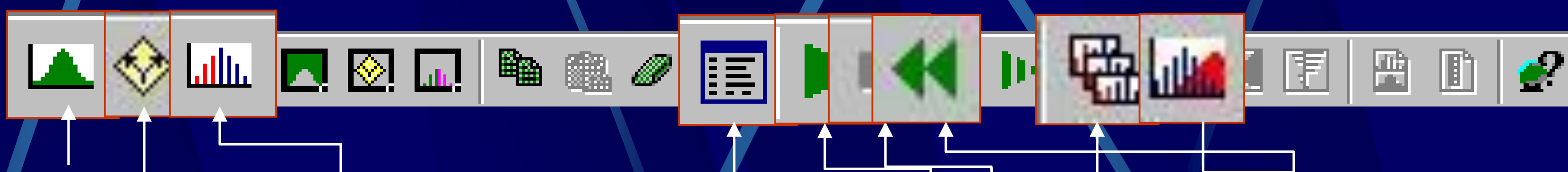
- Two new manus and a new toolbar are added to the Excel screen:





Conducting Simulation using Crystal Ball

- The new Toolbar is:



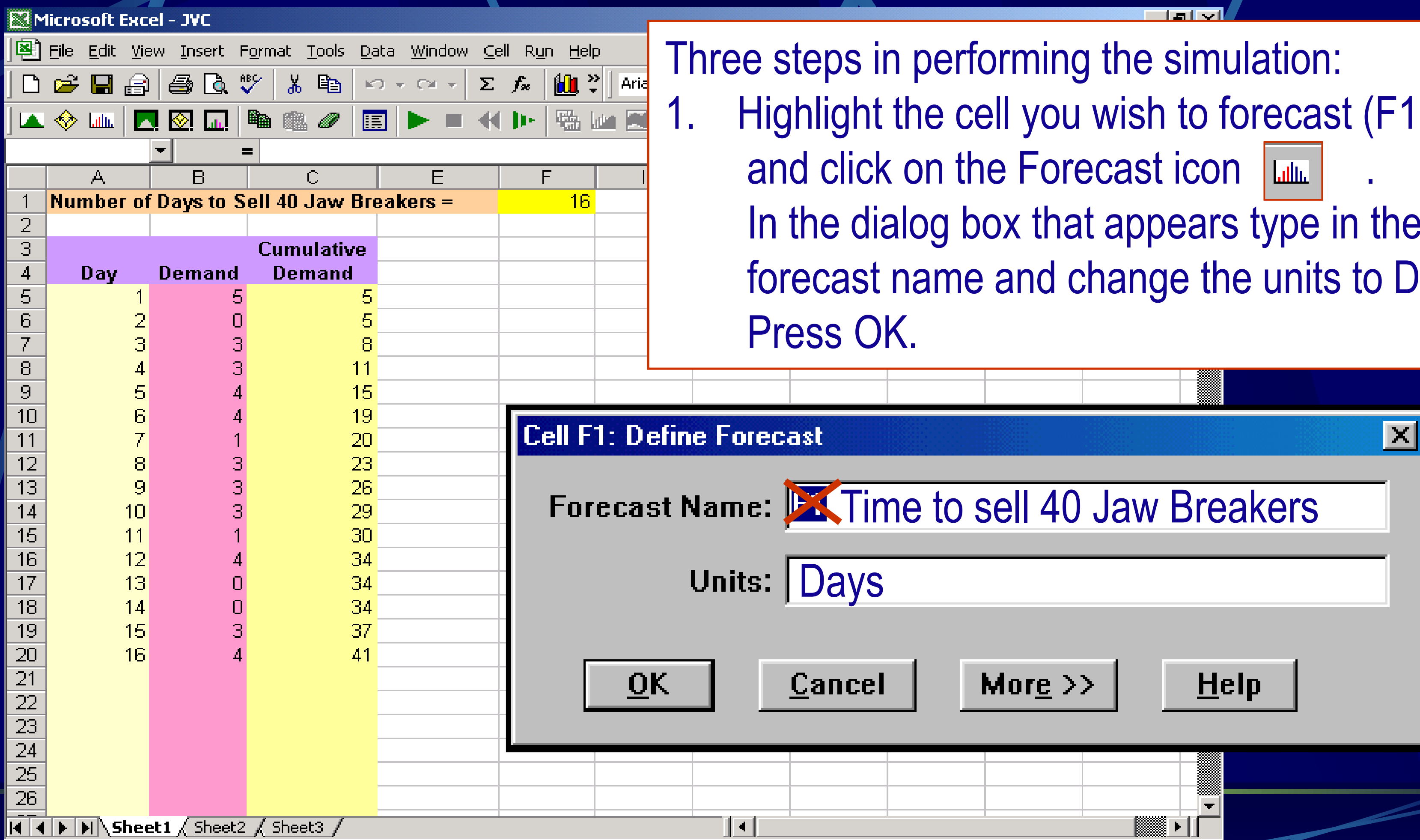
Hypothesis Testing with Crystal Ball-
Revisit the Bill Jewel Vending Machine Problem

- Recall: Bill wishes to determine the average number of days it will take to sell 40 or more jaw-breakers.
- The file JVC.xls contains the simulation run for one filling of the vending machine.

Hypothesis Testing with Crystal Ball-
Revisit the Bill Jewel Vending Machine Problem

Three steps in performing the simulation:

1. Highlight the cell you wish to forecast (F1), and click on the Forecast icon .
   In the dialog box that appears type in the forecast name and change the units to Days.
   Press OK.

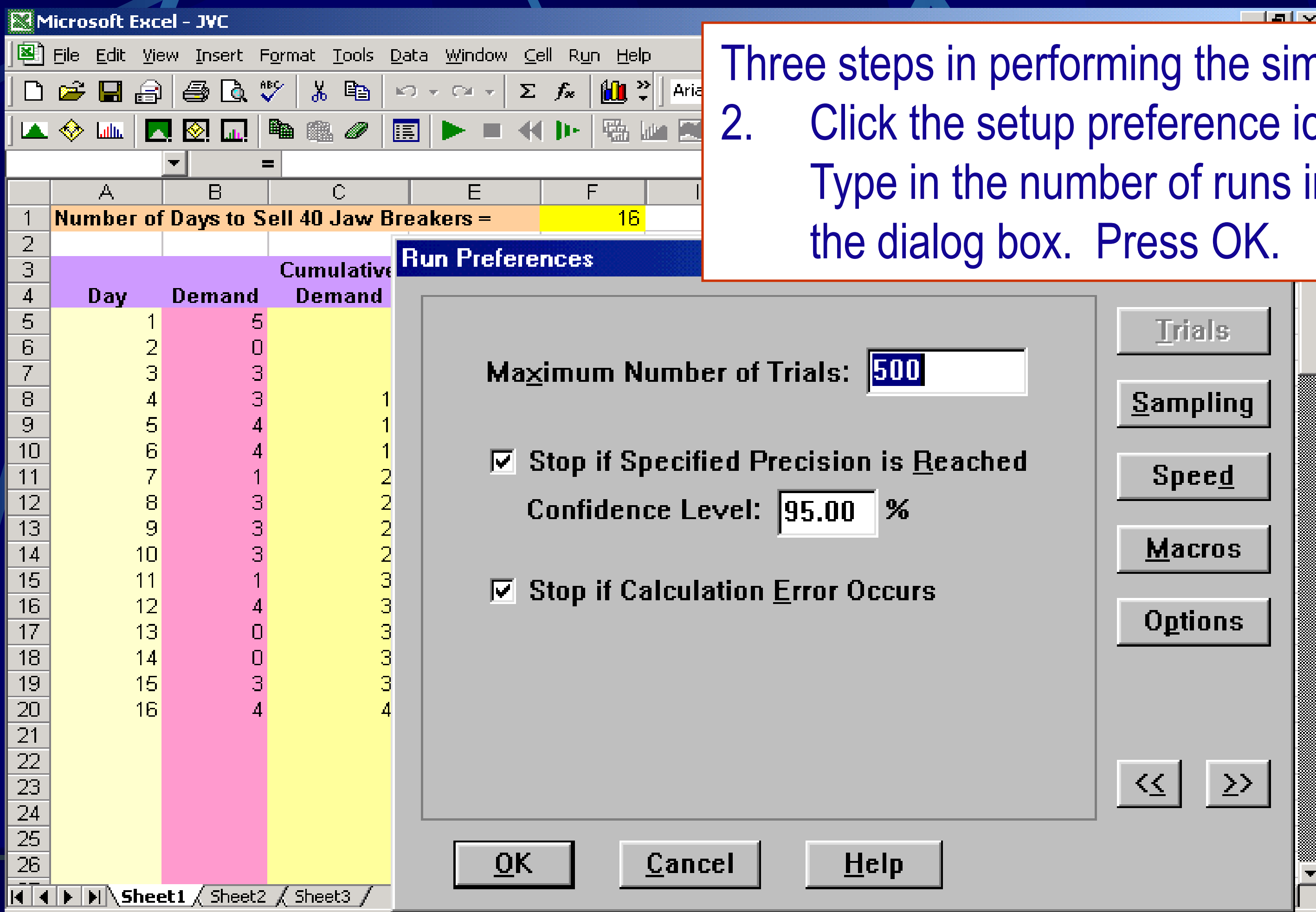


Day	Demand	Cumulative Demand
1	5	5
2	0	5
3	3	8
4	3	11
5	4	15
6	4	19
7	1	20
8	3	23
9	3	26
10	3	29
11	1	30
12	4	34
13	0	34
14	0	34
15	3	37
16	4	41

Hypothesis Testing with Crystal Ball-
Revisit the Bill Jewel Vending Machine Problem

Three steps in performing the simulation:

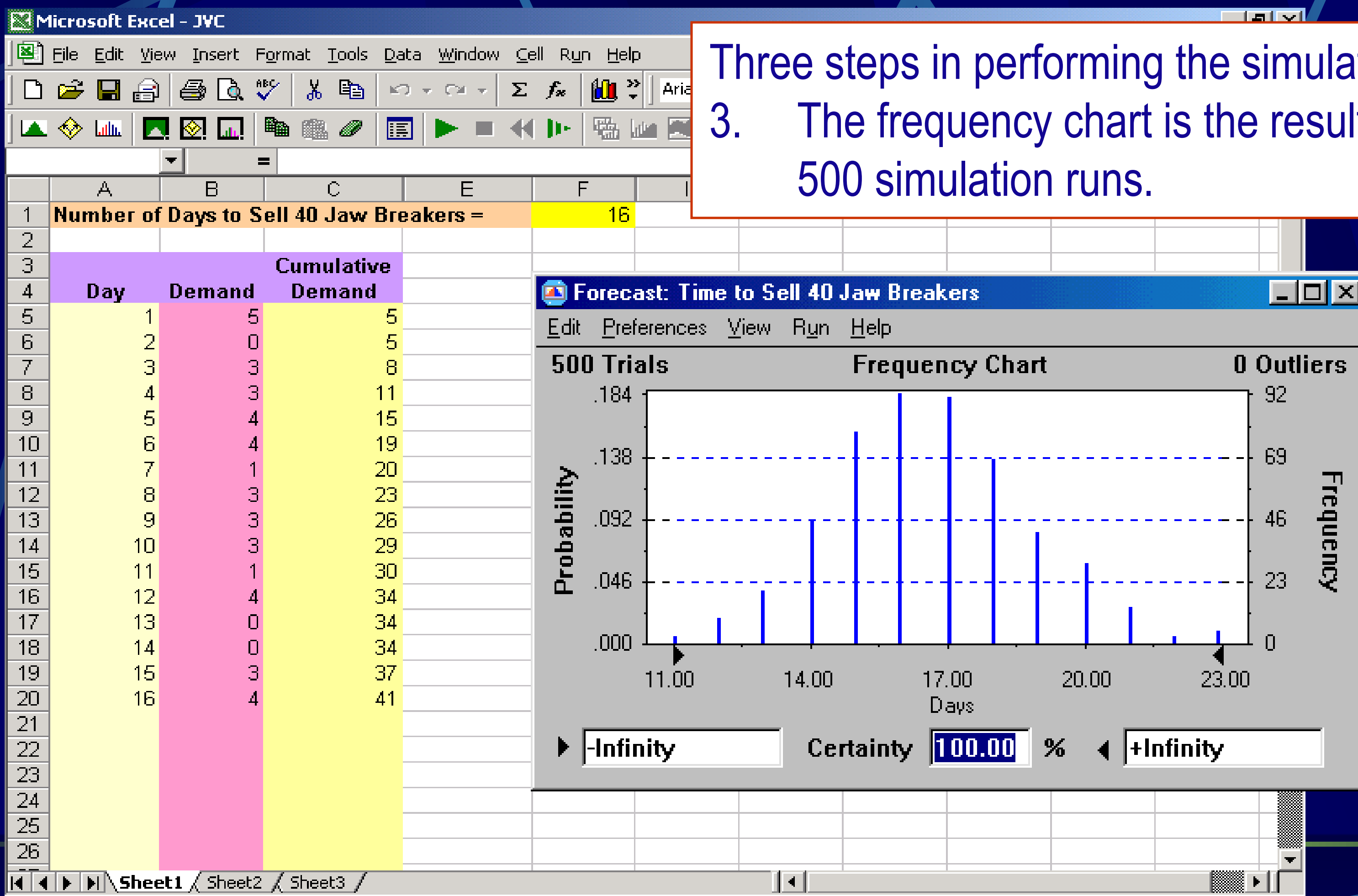
2. Click the setup preference icon Type in the number of runs in the dialog box. Press OK.



Hypothesis Testing with Crystal Ball- Revisit the Bill Jewel Vending Machine Problem

Three steps in performing the simulation:

3. The frequency chart is the result of 500 simulation runs.



Hypothesis Testing with Crystal Ball- Revisit the Bill Jewel Vending Machine Problem

- The hypothesis test is:
  - H_0: $\mu = 16$
    H_A: $\mu \neq 16$
  - From the frequency chart it appears the times follow a normal distribution.
  - We can use the t-distribution to test the above hypotheses.

Hypothesis Testing with Crystal Ball-
Revisit the Bill Jewel Vending Machine Problem

- From View>Statistics we get the following results:

  Mean = 16;   Standard error = .10

$$t = \frac{mean - 16}{standard\ \ error} = \frac{16.62 - 16}{.10} = 6.2$$

With a sample of 500 (499 degrees of freedom), we can use the Z value to conduct the test. The value of 6.2 is large enough to reject the null hypothesis for any reasonable significance level.

Confidence Interval with Crystal Ball- Revisit the Bill Jewel Vending Machine Problem

- We repeat the experiment for another 5000 days. The statistics of this experiment are:

  Mean = 16.44 Standard error = .03

- The confidence interval is:

$$\{\bar{X} - t_{\alpha/2,n-1} s/\sqrt{n},\ \bar{X} + t_{\alpha/2,n-1} s/\sqrt{n}\}$$

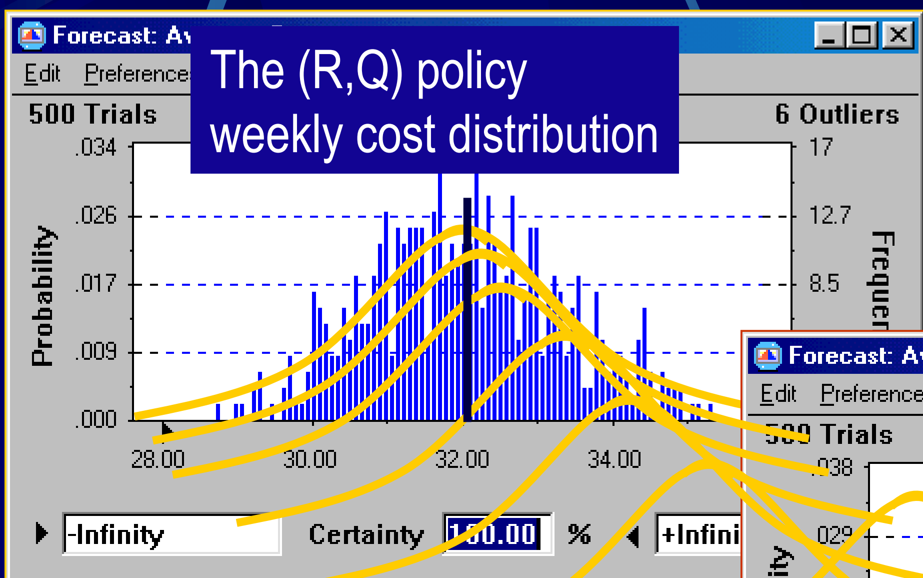
This results in: {16.44±1.96(.03)} = {16.38, 16.50}.

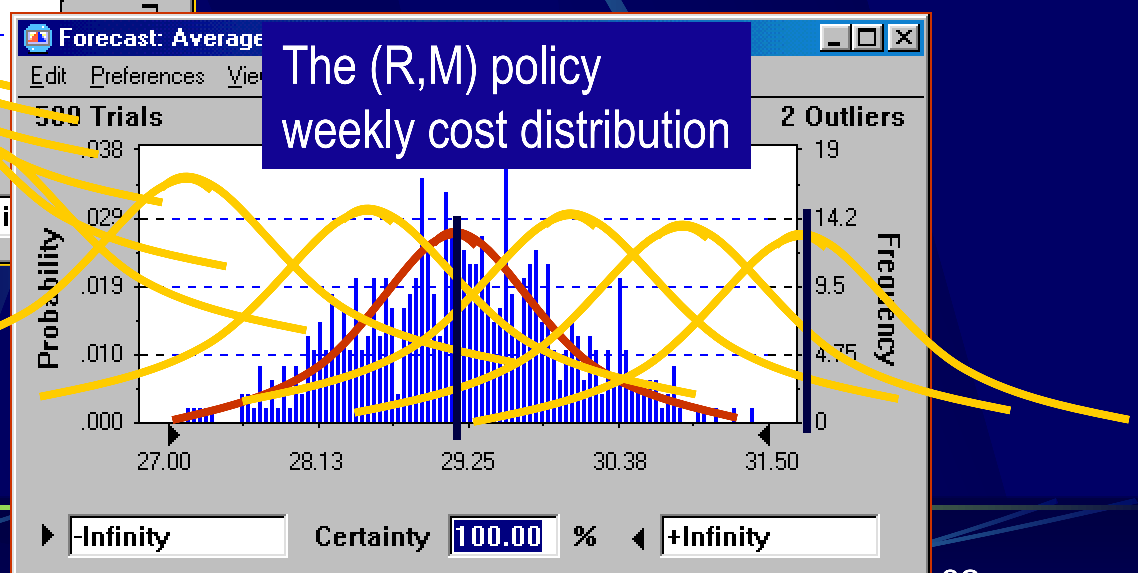
Determining an Inventory Policy– Revisiting Allen Appliances Comp.

- We compare a (R,M) policy to the (R,Q) policy studied previously.
- The cell (J11) that calculates the fixed order Q^* for the simulation of the (R,Q) policy, is changing to $Q^* + (R – I)$ for the simulation of the (R,M) policy.
- The comparative study results are shown next.

Comparing the (R,Q) and (R,M) Inventory Policies

The simulation provides also a summary of statistical results. Click.





Comparing the (R,Q) and (R,M) Inventory Policies

Forecast: Average Cost =

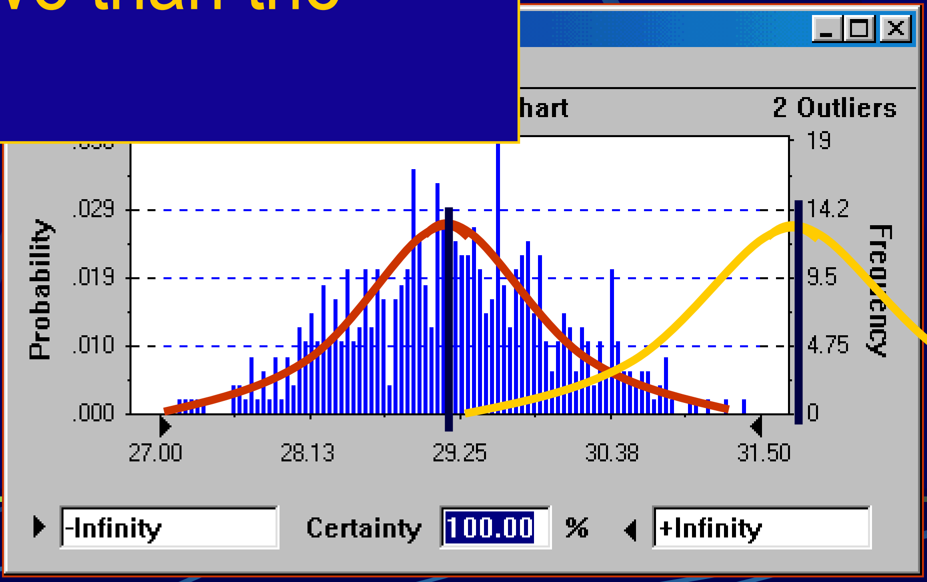
Edit Preferences View Run Help

Cell B7 **Statistics**

Statistic	Value (R,Q)	Value (R,M)
Trials	500	500
Mean	32.14	29.22
Median	32.05	29.21
Mode	32.89	28.41
Standard Deviation	1.40	0.79
Variance	1.97	0.63
Skewness	0.50	0.12
Kurtosis	3.55	2.93
Coeff. of Variability	0.04	0.03
Range Minimum	28.74	27.17
Range Maximum	37.67	32.01
Range Width	8.93	4.84
Mean Std. Error	0.06	0.04

Comparing the (R,Q) and (R,M) Inventory Policies

Is there sufficient evidence in the simulated data to infer that the (R,M) policy is less expensive than the (R,Q) policy?



Comparing the (R,Q) and (R,M) Inventory Policies

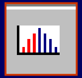
- Hypothesis test
  - H_0: $\mu_1 - \mu_2 = 0$
    H_A: $\mu_1 - \mu_2 > 0$
  - The rejection region in standard terms: $Z > Z_\alpha$.
  - The Z statistic is

Population 1 – the (R,Q) policy weekly cost
Population 2 - the (R,M) policy weekly cost

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n} + \frac{s_2^2}{n}}} = \frac{32.14 - 29.22}{\sqrt{\frac{1.97}{500} + \frac{.63}{500}}} = 23.69$$

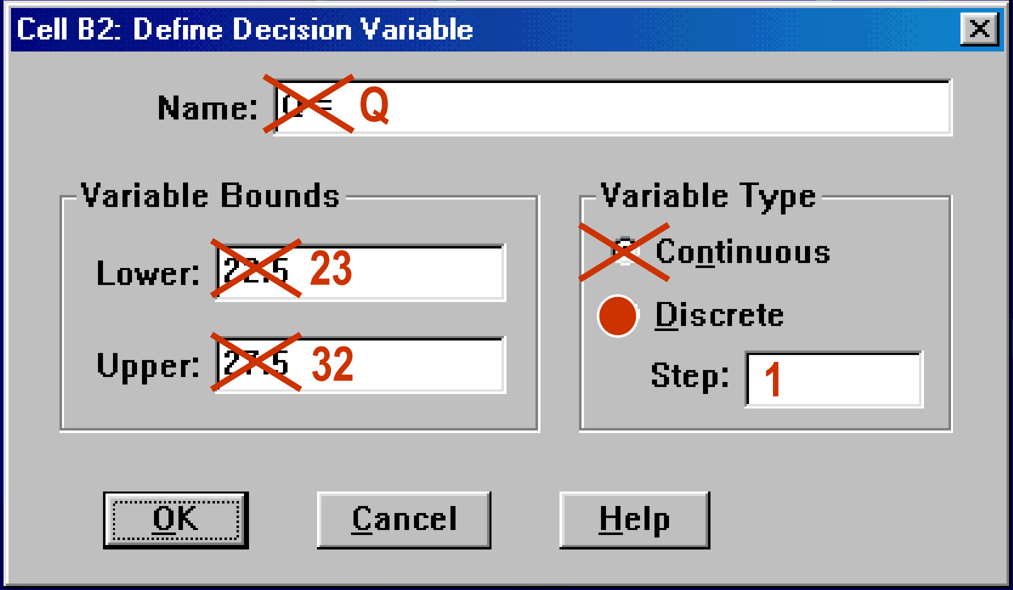
We have a clear evidence that the (R,M) policy is cheaper.

Finding the "Best" (R,M) policy

- Let us run the simulation for various values of Q between 23 and 32.
- We create a decision table in Crystal Ball as follows:
  - Reset the simulation 
  - Define the average cost (B7) as the forecast value (highlight cell B7 and click ).
  - Define the order quantity as the decision variable (highlight cell B2 and click ).

Finding the "Best" (R,M) policy
Creating a decision table in Crystal Ball

- In the dialog box that appears make the following changes:



Finding the "Best" (R,M) policy

Creating a decision table in Crystal Ball - continued

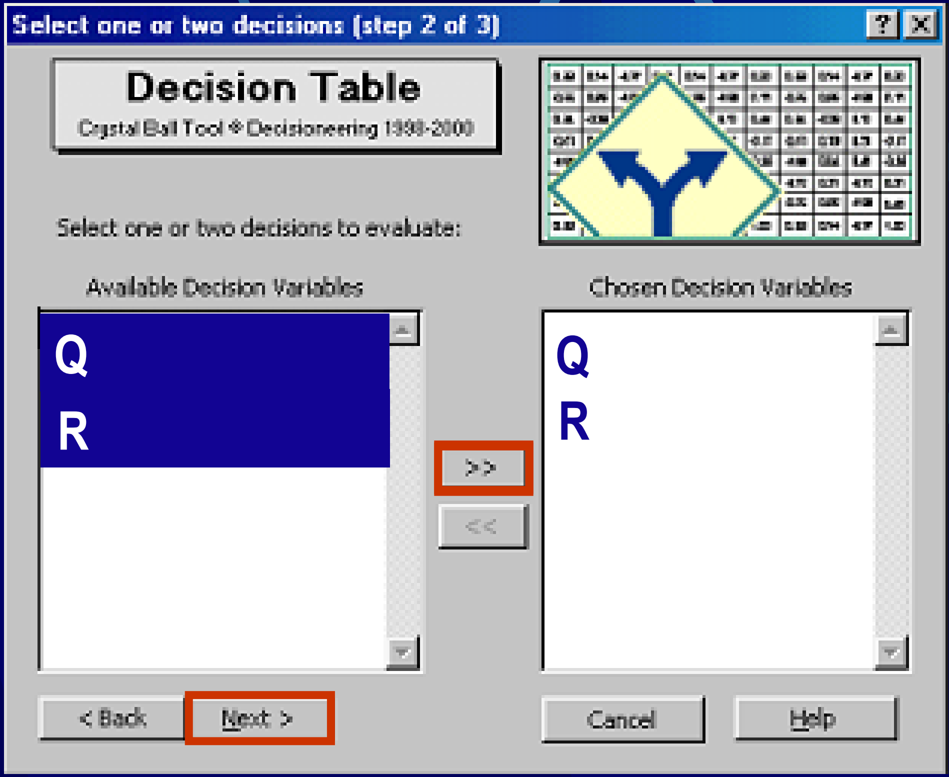
- Define the reorder point (B3) as a decision variable
  - (Highlight cell B3 and click on ).
  - On the dialog that appears we make the changes:
    - Name of the decision variable: R (delete R=)
    - Lower bound = 8
    - Upper bound = 17
    - Variable type = Discrete
    - Step = 1

Finding the "Best" (R,M) policy
Creating a decision table in Crystal Ball - continued

- Setup the decision table
  - Select Decision Table from the CBTools menu bar. In the dialog box that appears click "Next" (because "Average Cost =" is already highlighted).
  - In the dialog box that appears select each variable (Q and then R and move each one to the right hand list (by clicking the button >>).

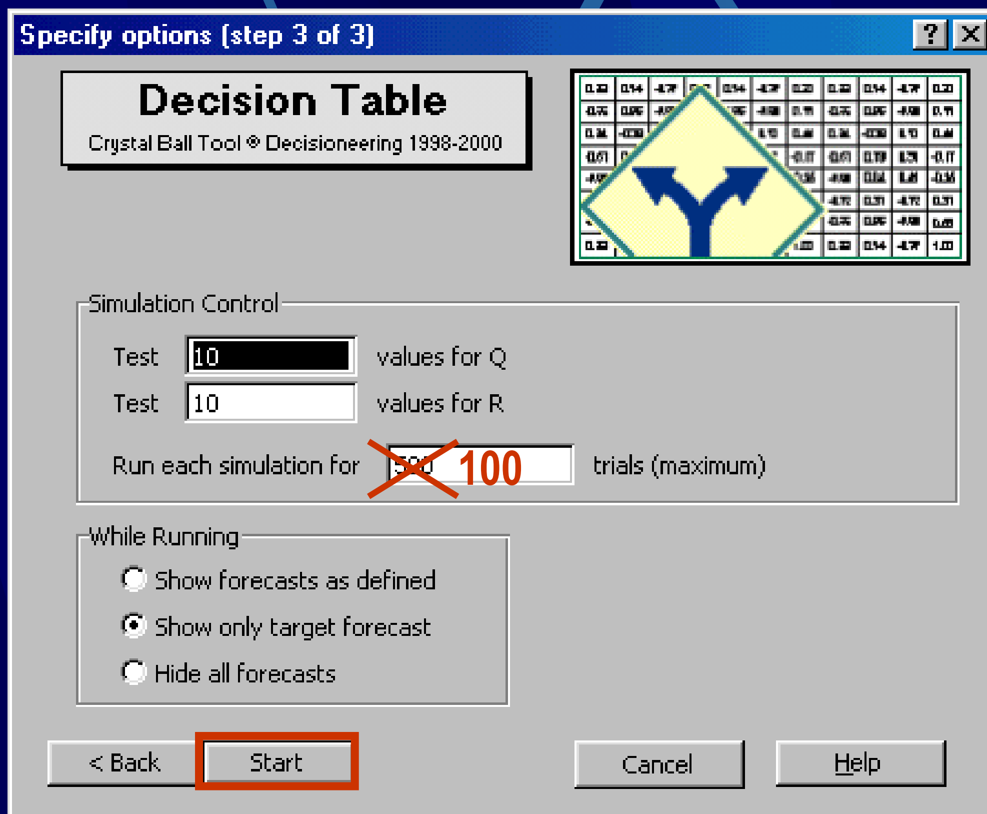
Finding the "Best" (R,M) policy

Creating a decision table in Crystal Ball - continued



Finding the “Best” (R,M) policy

Creating a decision table in Crystal Ball - continued



Finding the "Best" (R,M) policy

Analyzing the Simulation Results

	A	B	C	D	E	F	G	H	I	J	K	L
1	Trend Overlay	Q (23)	Q (24)	Q (25)	Q (26)	Q (27)	Q (28)	Q (29)	Q (30)	Q (31)	Q (32)	
2	R (8)	32.49628	32.0207	31.50864	31.2252	30.78742	30.57182	30.12943	30.11337	29.97291	29.80672	1
3	R (9)	31.18105	30.61218	30.34156	29.893	29.6994	29.46378	29.14451	29.2245	29.01147	28.99288	2
4	R (10)	29.98043	29.51874	29.19553	28.90977	28.74925	28.58188	28.48685	28.4306	28.23725	28.31075	3
5	R (11)	28.99812	28.58733	28.34429	28.14743	27.92502	27.89588	27.87937	27.6989	27.77291	27.7307	4
6	R (12)	28.25184	27.88557	27.62985	27.4							
7	R (13)	27.4851	27.30369	27.17235	26.9							
8	R (14)	27.05469	26.89896	26.76435	26.7							
9	R (15)	26.82684	26.67352	26.57487	26.6							
10	R (16)	26.72675	26.69879	26.60806	26.6							
11	R (17)	26.72559	26.68186	26.69509	26.8							
12		1	2	3								
13												
14												

Let us refine the search for the optimal policy. Around the point (Q = 25, R = 14 we perform more simulations with 500 runs per each pair of Q and R.

Finding the “Best” (R,M) policy
Analyzing the Simulation Results

	A	C	D	E	F	G	H
1	Trend Chart Overlay Chart Forecast Charts	Q (24)	Q (25)	Q (26)	Q (27)		
2	R (13)	27.357762	27.145664	27.074616	26.966922	1	
3	R (14)	26.886306	26.79954	26.742982	26.76362	2	
4	R (15)	26.668446	26.610806	26.598364	26.693134	3	
5	R (16)	26.657282	26.641588	26.653122	26.72946	4	
6	R (17)	26.737086	26.736996	26.813514	26.879124	5	
7		2	3	4	5		
8							
9							

“The “best” (R,M) inventory policy should be based on Q = 25, or 26 and R = 15.