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PUSAT MASSA

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Pusat Massa

Istilah "pusat massa" dan "titik berat" digunakan secara sinonim dalam medan gravitasi seragam untuk mewakili titik tunggal dalam suatu objek atau sistem yang dapat digunakan untuk menggambarkan respon sistem untuk gaya dan torsi eksternal.

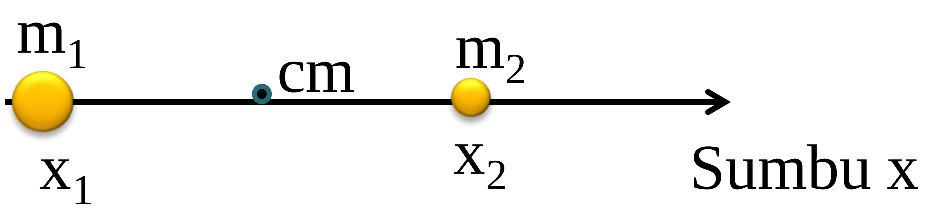
Pusat Massa

Perhatikan gerakan pusat massa dari seorang penari balet

Sumber: The Physics of Dance, Kenneth

Pusat Massa untuk Partikel Diskrit

Pusat massa adalah titik di mana semua massa dianggap "terkonsentrasi" di titik tersebut.



$$(\mathbf{m_1} + \mathbf{m_2})\, \mathbf{x_{cm}} = \mathbf{m_1}\, \mathbf{x_1} + \mathbf{m_2}\, \mathbf{x_2}$$

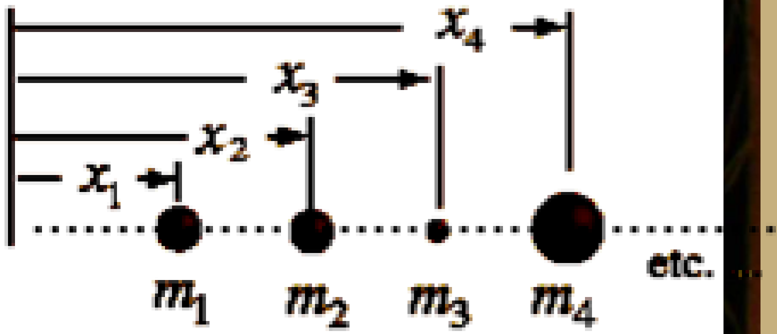
Massa total

Pusat massa

Jumlah momen masing masing massa

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Untuk yang lebih umum :



$$Mx_{cm} = \sum_{i=1}^{N} m_i x_i \quad \text{where} \quad M = \sum_{i=1}^{N} m_i = \text{total mass}$$

$$x_{cm} = \frac{\sum_{i=1}^{N} m_i x_i}{M}$$

Dalam sistem koordinat ruang :

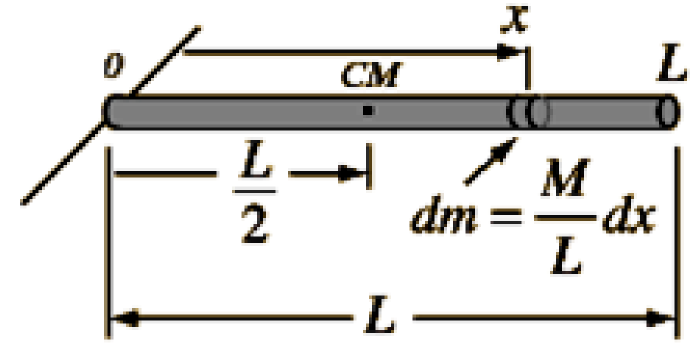
$$x_{cm} = \frac{\sum_{i=1}^{N} m_i x_i}{M} \qquad y_{cm} = \frac{\sum_{i=1}^{N} m_i y_i}{M} \qquad z_{cm} = \frac{\sum_{i=1}^{N} m_i z_i}{M}$$

Untuk massa kontinyu :

$$x_{cm} = \lim_{\Delta m \to 0} \frac{\sum_{i=1}^{N} \Delta m_i x_i}{M} = \frac{\int_0^M x\,dm}{M}$$

Untuk kasus batang seragam / uniform :

$$x_{cm} = \frac{\int_0^L x \frac{M}{L} dx}{M} = \frac{1}{L} \frac{x^2}{2} \Bigg|_{x=0}^{x=L} = \frac{L}{2}$$



(A) Show that the center of mass of a rod of mass M and length L lies midway between its ends, assuming the rod has a uniform mass per unit length.

Solution The rod is shown aligned along the x axis in Figure 9.22, so that $y_{CM} = z_{CM} = 0$. Furthermore, if we call the mass per unit length λ (this quantity is called the *linear mass density*), then $\lambda = M/L$ for the uniform rod we assume here. If we divide the rod into elements of length dx, then the mass of each element is $dm = \lambda\, dx$. Equation 9.31 gives

$$x_{CM} = \frac{1}{M}\int x\, dm = \frac{1}{M}\int_0^L x\lambda\, dx = \frac{\lambda}{M}\frac{x^2}{2}\bigg|_0^L = \frac{\lambda L^2}{2M}$$

Because $\lambda = M/L$, this reduces to

$$x_{CM} = \frac{L^2}{2M}\left(\frac{M}{L}\right) = \frac{L}{2}$$

One can also use symmetry arguments to obtain the same result.

(B) Suppose a rod is *nonuniform* such that its mass per unit length varies linearly with x according to the expression $\lambda = \alpha x$, where α is a constant. Find the x coordinate of the center of mass as a fraction of L.

Solution In this case, we replace dm by $\lambda\, dx$, where λ is not constant. Therefore, x_{CM} is

$$x_{CM} = \frac{1}{M}\int x\, dm = \frac{1}{M}\int_0^L x\lambda\, dx = \frac{1}{M}\int_0^L x\alpha x\, dx$$

$$= \frac{\alpha}{M}\int_0^L x^2\, dx = \frac{\alpha L^3}{3M}$$

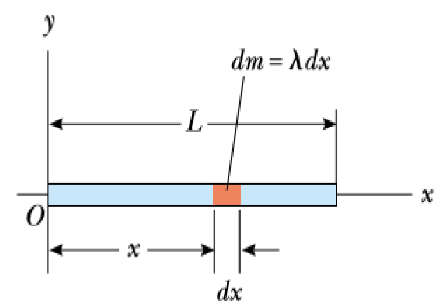
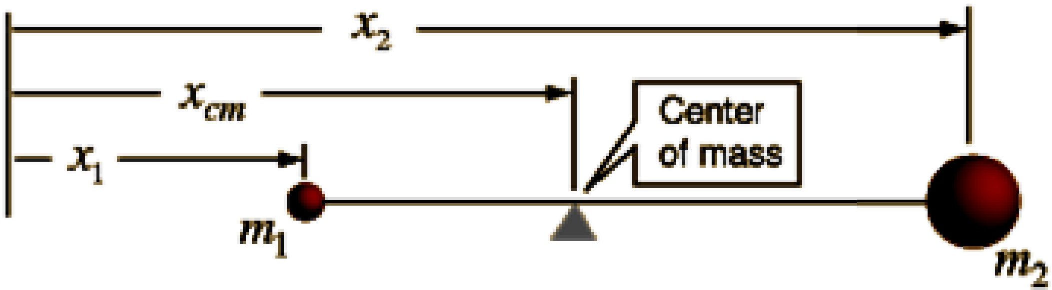


Figure 9.22 (Example 9.14) The geometry used to find the center of mass of a uniform rod.

Materi

Sistem "jungkat-jungkit" akan dalam keadaan setimbang bila titik tumpuan berada pusat massanya, dan ini berhubungan dengan momen (torsi) massa.



$$m_1 r_1 = m_2 r_2 \qquad m_1 = m_2 \frac{r_2}{r_1}$$

Dua beanda bermassa 5 kg dan 3 kg berada pada posisi x = 2 m dan 4 m dari titik acuan. Dimana letak titik pusat massa dari sistem dua benda tersebut ?.

Penyelesaian :

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$x_{cm} = \frac{5.2 + 3.4}{5 + 3} = 2{,}625\, m$$

Three particles of masses m_1 = 1.2 kg, m_2 = 2.5 kg, and m_3 = 3.4 kg form an equilateral triangle of edge length *a 140 cm. Where is the center of mass of this system?*

Penyelesaian :

Particle	Mass (kg)	x (cm)	y (cm)
1	1.2	0	0
2	2.5	140	0
3	3.4	70	120

The total mass *M* of the system is 7.1 kg.

$$x_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i x_i = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{M}$$

$$= \frac{(1.2\text{ kg})(0) + (2.5\text{ kg})(140\text{ cm}) + (3.4\text{ kg})(70\text{ cm})}{7.1\text{ kg}}$$

$$= 83\text{ cm} \qquad \text{(Answer)}$$

and $$y_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i y_i = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{M}$$

$$= \frac{(1.2\text{ kg})(0) + (2.5\text{ kg})(0) + (3.4\text{ kg})(120\text{ cm})}{7.1\text{ kg}}$$

$$= 58\text{ cm}. \qquad \text{(Answer)}$$

In Fig. 9-4, the center of mass is located by the position vector $\vec{r}_{com}$, which has components x_{com} and y_{com}.

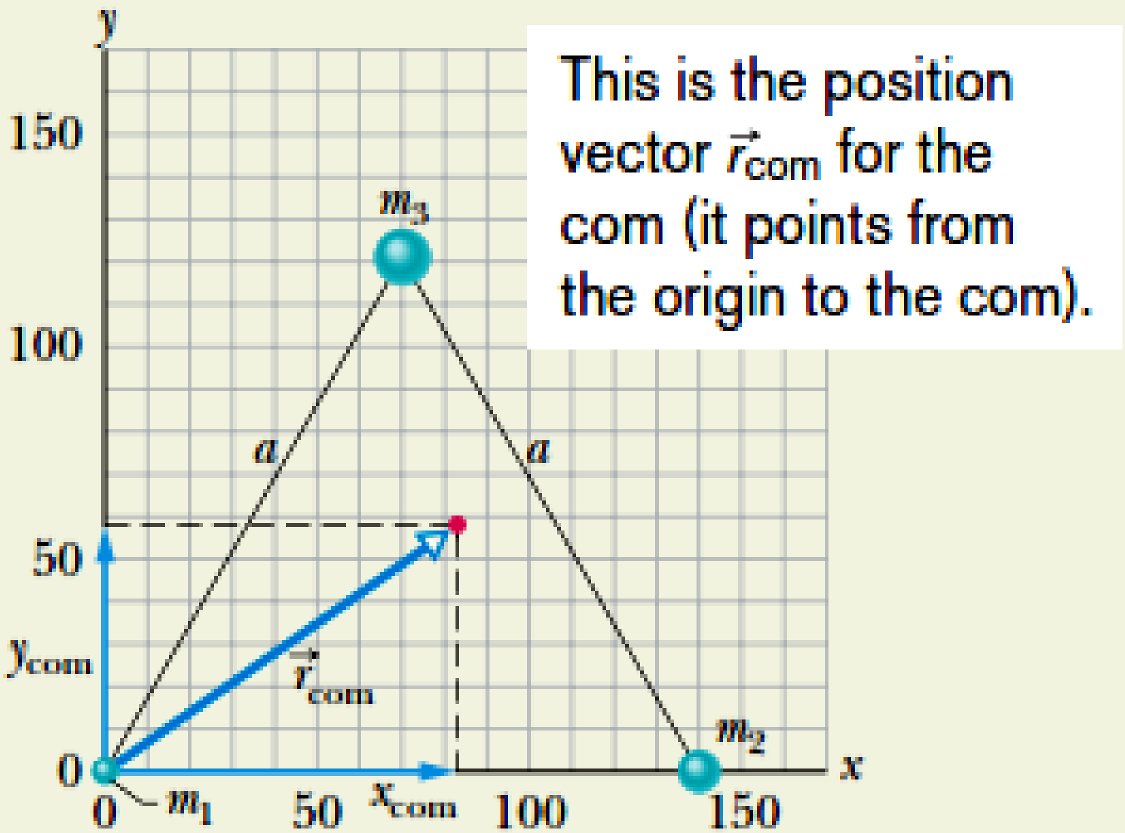


Fig. 9-4 Three particles form an equilateral triangle of edge length a. The center of mass is located by the position vector $\vec{r}_{com}$.

Ringkasan

Pusat massa adalah titik di mana semua massa atau elemen massa dianggap "terkonsentrasi” di titik tersebut.

LATIHAN SOAL

•1 A 2.00 kg particle has the xy coordinates $(-1.20\text{ m}, 0.500\text{ m})$, and a 4.00 kg particle has the xy coordinates $(0.600\text{ m}, -0.750\text{ m})$. Both lie on a horizontal plane. At what (a) x and (b) y coordinates must you place a 3.00 kg particle such that the center of mass of the three-particle system has the coordinates $(-0.500\text{ m}, -0.700\text{ m})$?

•2 Figure 9-35 shows a three-particle system, with masses m_1 = 3.0 kg, m_2 = 4.0 kg, and m_3 = 8.0 kg. The scales on the axes are set by x_s = 2.0 m and y_s = 2.0 m. What are (a) the x coordinate and (b) the y coordinate of the system's center of mass? (c) If m_3 is gradually increased, does the center of mass of the system shift toward or away from that particle, or does it remain stationary?

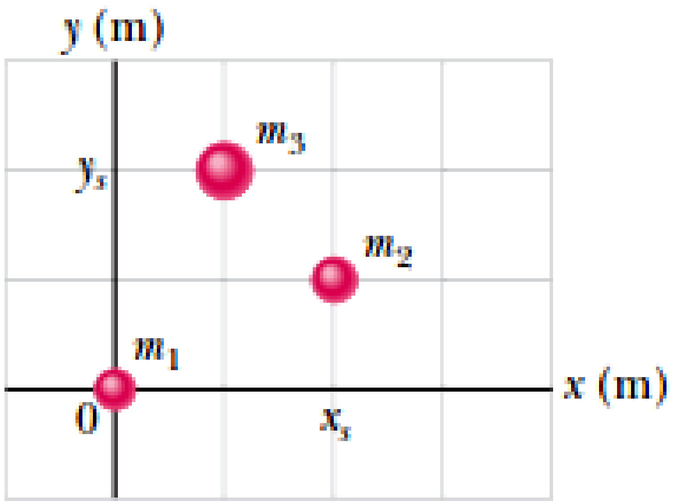


Fig. 9-35 Problem 2.

••3 Figure 9-39 shows a cubical box that has been constructed from uniform metal plate of negligible thickness. The box is open at the top and has edge length $L = 40$ cm. Find (a) the x coordinate, (b) the y coordinate, and (c) the z coordinate of the center of mass of the box.

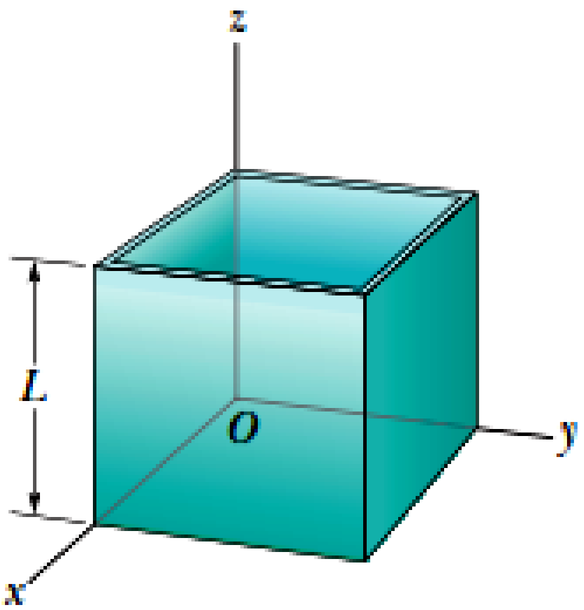


Fig. 9-39 Problem 6.

SEKIAN
&
TERIMAKASIH