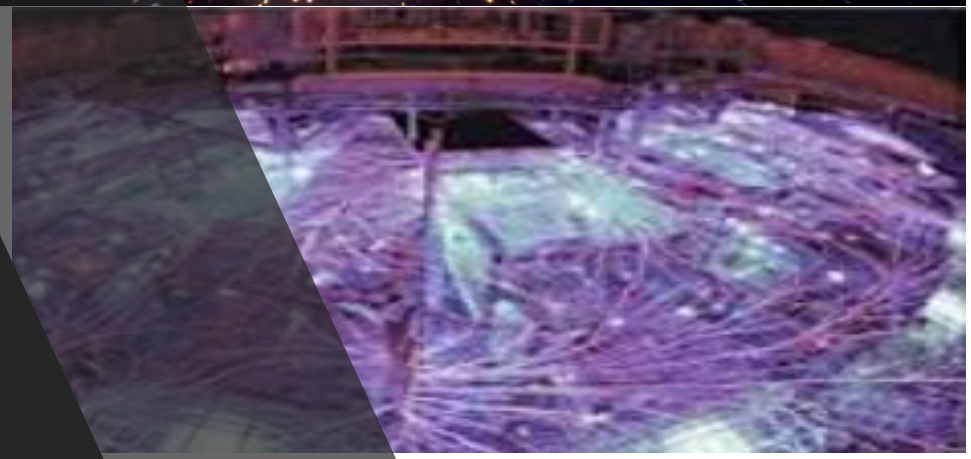
PERSAMAAN DIFFERENSIAL

PD Bernoulli

MATEMATIKA REKAYASA 1

AULIA SITI AISJAH – TEKNIK FISIKA ITS



1. BENTUK STANDAR PD BERNOULLI

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

Peny PD Bernoulli, dengan cara gantikan y menjadi z, dimana z = $z = y^{1-n}$

$$\frac{1}{y^n}\frac{dy}{dx} + P(x)y^{1-n} = Q(x)$$

$$\frac{1}{(1-n)}\frac{dz}{dx} + P(x)z = Q(x)$$

Bentuk Umum PD Bernoulli

$$\frac{dy}{dx} + P(x)y = Q(x)\,y^n$$

Gunakan $z = y^{1-n}$ untuk menjadi PD Bernoulli menjadi PD linier

Selesaikan dengan menggunakan Faktor Pengintegrasi

Contoh PD Bernoulli

$\frac{dy}{dx} - \frac{1}{x}y = xy^2$	$2\frac{dy}{dx} + \tan x \cdot y = \frac{(4x+5)^2}{\cos x}y^3$
$\frac{dy}{dx} + \frac{y}{x} = y^2$	$x\frac{dy}{dx} + y = y^2x^2 \ln\, x$
$\frac{dy}{dx} + \frac{1}{3}y = e^x y^4$	$\frac{dy}{dx} = y\cot x + y^3 \mathrm{cosec} x$
$x\frac{dy}{dx} + y = xy^3$	
$\frac{dy}{dx} + \frac{2}{x}y = -x^2 \cos x \cdot y^2$	

Faktor Pengintegrasi

$$\boxed{\frac{dz}{dx} + P_1(x)z = Q_1(x)}$$

$$\text{IF} = e^{\int P_1(x)dx}$$

$$\frac{d}{dx}(\text{IF}\ z) = \text{IF}\ Q_1(x)$$

$$\text{IF}\ z = \int \text{IF}\ Q_1(x)\, dx$$

Contoh

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

$$\frac{1}{y^n}\frac{dy}{dx} + P(x)y^{1-n} = Q(x)$$

$$\frac{dz}{dx} = (1-n)y^{(1-n-1)}\frac{dy}{dx}$$

$$\frac{1}{(1-n)}\frac{dz}{dx} = \frac{1}{y^n}\frac{dy}{dx}$$

$$\frac{1}{(1-n)}\frac{dz}{dx} + P(x)z = Q(x)$$

$$\boxed{\frac{dz}{dx} + P_1(x)z = Q_1(x)}$$

$$P_1(x) = (1-n)P(x)$$
$$Q_1(x) = (1-n)Q(x)\ .$$

Contoh

$$\frac{dy}{dx} + P(x)y = Q(x)y^n \text{ where}$$

$$\begin{aligned} P(x) &= -\frac{1}{x} \\ Q(x) &= x \\ n &= 2 \end{aligned}$$

Dibagi dengan y^n:

$$\frac{1}{y^2}\frac{dy}{dx} - \frac{1}{x}y^{-1} = x$$

$z = y^{1-n} = y^{-1}$:

$$\frac{dz}{dx} = -y^{-2}\frac{dy}{dx} = -\frac{1}{y^2}\frac{dy}{dx}$$

$$-\frac{dz}{dx} - \frac{1}{x}z = x$$

$$\frac{dz}{dx} + \frac{1}{x}z = -x$$

$$\frac{dz}{dx} + \frac{1}{x}z = -x$$

Faktor Pengintegrasi

$$\text{IF} = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$$

$$x\frac{dz}{dx} + z = -x^2$$

$$\frac{d}{dx}[x \cdot z] = -x^2$$

$$xz = -\int x^2 dx$$

$$xz = -\frac{x^3}{3} + C$$

$$\frac{x}{y} = -\frac{x^3}{3} + C$$

$$\frac{1}{y} = -\frac{x^2}{3} + \frac{C}{x}\,.$$

Gunakan $z = \frac{1}{y}$:

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

dimana $P(x) = \frac{1}{x}$,

$Q(x) = 1$,

$n = 2$

set $\underline{z = y^{1-n} = y^{-1}}$

$$\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{x}y^{-1} = 1$$

$$\frac{dz}{dx} = -1 \cdot y^{-2}\frac{dy}{dx} = -\frac{1}{y^2}\frac{dy}{dx}$$

$$-\frac{dz}{dx} + \frac{1}{x}z = 1$$

$$\frac{dz}{dx} - \frac{1}{x}z = -1$$

Faktor Pengintegrasi

$$\text{IF} = e^{-\int \frac{dx}{x}} = e^{-\ln x} = e^{\ln x^{-1}} = \frac{1}{x}$$

$$\frac{1}{x}\frac{dz}{dx} - \frac{1}{x^2}z = -\frac{1}{x}$$

$$\frac{d}{dx}\left[\frac{1}{x} \cdot z\right] = -\frac{1}{x}$$

$$\frac{1}{x} \cdot z = -\int \frac{dx}{x}$$

$$\frac{z}{x} = -\ln x + C$$

$$\frac{1}{yx} = C - \ln x$$

$$\frac{1}{y} = x(C - \ln x)\ .$$

Faktor Pengintegrasi

$\frac{dy}{dx} + P(x)y = Q(x)y^n$ dimana

$$\begin{aligned} P(x) &= \frac{1}{3} \\ Q(x) &= e^x \\ n &= 4 \end{aligned}$$

$$\frac{1}{y^4}\frac{dy}{dx} + \frac{1}{3}y^{-3} = e^x$$

Bagi dengan $\underline{y^n}$:

$$\frac{dz}{dx} = -3y^{-4}\frac{dy}{dx} = -\frac{3}{y^4}\frac{dy}{dx}$$

set $\underline{z = y^{1-n} = y^{-3}}$:

$$-\frac{1}{3}\frac{dz}{dx} + \frac{1}{3}z = e^x$$

$$\frac{dz}{dx} - z = -3e^x$$

$$\mathrm{IF} = e^{-\int dx} = e^{-x}$$

$$e^{-x}\frac{dz}{dx} - e^{-x}z = -3e^{-x}\cdot e^x$$

$$\frac{d}{dx}[e^{-x}\cdot z] = -3$$

$$e^{-x}\cdot z = \int -3\,dx$$

$$e^{-x}\cdot z = -3x + C$$

$$e^{-x}\cdot \tfrac{1}{y^3} = -3x + C$$

$$\frac{1}{y^3} = e^x(C - 3x)\,.$$

Pers. Bernoulli

$\frac{dy}{dx} + \frac{y}{x} = y^3$ dg $P(x) = \frac{1}{x}, Q(x) = 1, n = 3$

$$\frac{1}{y^3}\frac{dy}{dx} + \frac{1}{x}y^{-2} = 1$$

$$\frac{dz}{dx} = -2y^{-3}\frac{dy}{dx}$$

$$-\frac{1}{2}\frac{dz}{dx} = \frac{1}{y^3}\frac{dy}{dx}$$

$$-\frac{1}{2}\frac{dz}{dx} + \frac{1}{x}z = 1$$

$$\frac{dz}{dx} - \frac{2}{x}z = -2$$

Faktor Pengintegrasi

$$\text{IF} = e^{-2\int \frac{dx}{x}} = e^{-2\ln x} = e^{\ln x^{-2}} = \frac{1}{x^2}$$

$$\frac{1}{x^2}\frac{dz}{dx} - \frac{2}{x^3}z = -\frac{2}{x^2}$$

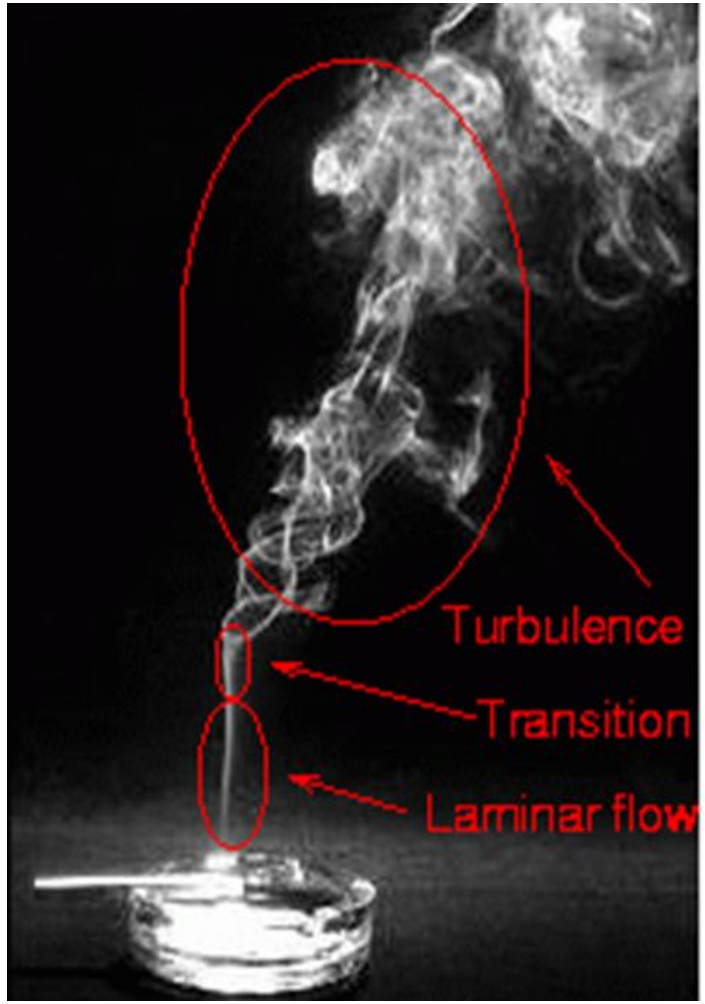
$$\frac{d}{dx}\left[\frac{1}{x^2}z\right] = -\frac{2}{x^2}$$

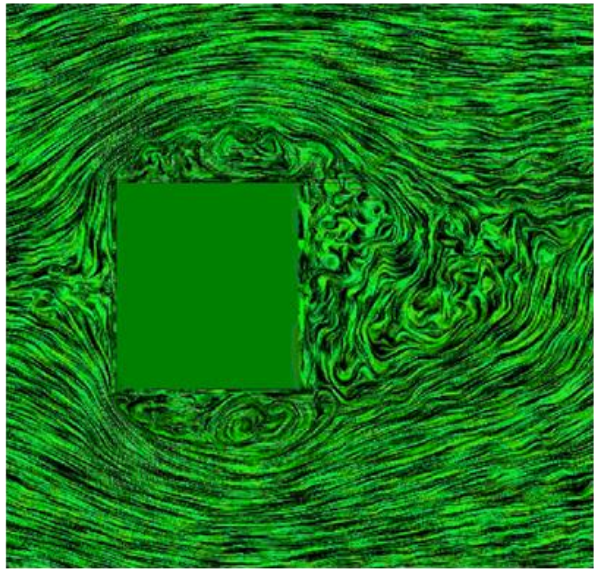
$$\frac{1}{x^2}z = (-2)\cdot(-1)\frac{1}{x} + C$$

$$z = 2x + Cx^2$$

$$y^2 = \frac{1}{2x+Cx^2}\ .$$

Comtoh – Aplikasi – Persamaan Bernoulli





Terimakasih