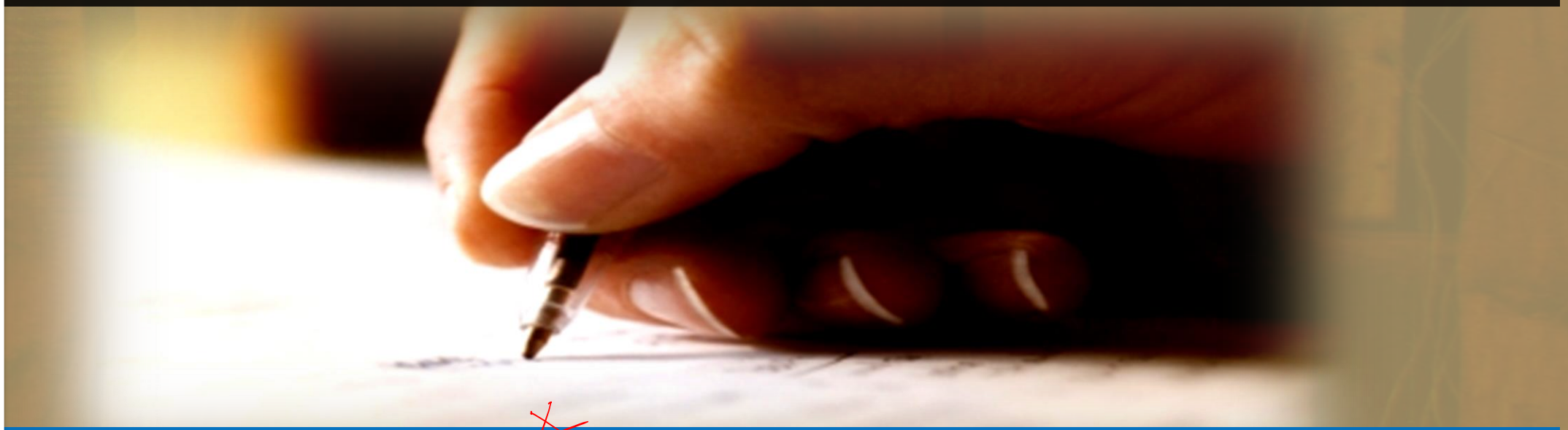
Institut Teknologi Sepuluh Nopember
Surabaya

DEPARTEMEN TEKNIK FISIKA - FTIRS



PENYELESAIN PERSAMAAN DIFERENSIAL DENGAN DERET PANGKAT

Seri: Matematika Rekayasa 1

Oleh: Aulia Siti Aisjah

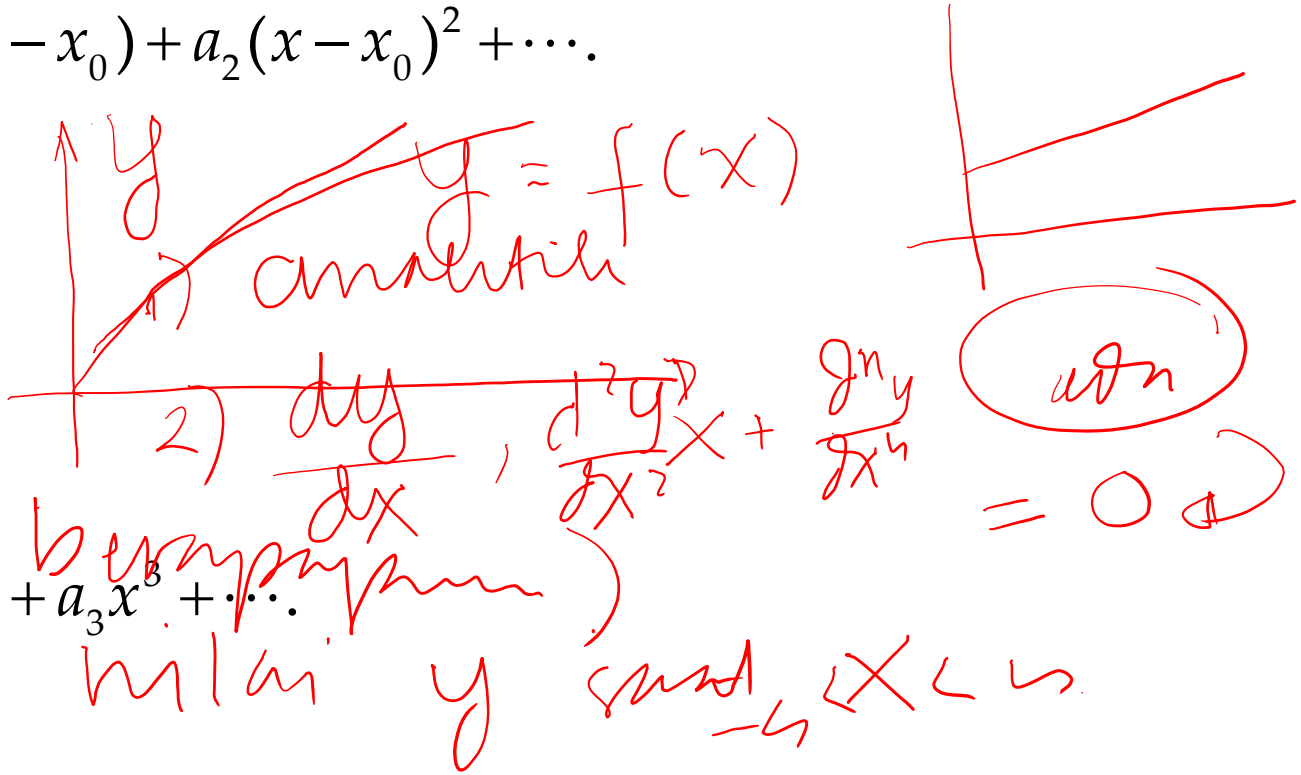
Deret Pangkat – bentuk khusus dari Deret Taylor

$$\sum_{m=0}^{\infty} a_m (x - x_0)^m = a_0 + a_1(x - x_0) + a_2(x - x_0)^2 + \cdots.$$

X0 = titik acuan

X0 = 0

$$\sum_{m=0}^{\infty} a_m x^m = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots.$$



Derect Taylor

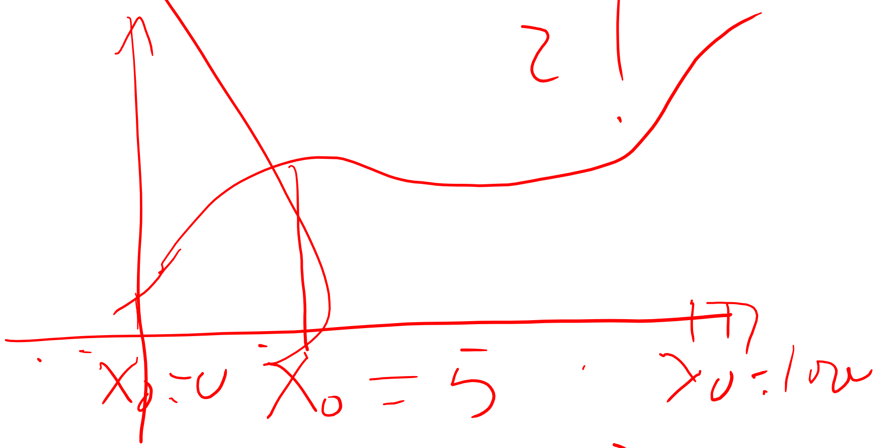
$y = f(x)$

$$= f(x=x_0) + \frac{f'(x=x_0)\,(x-x_0)}{1!} + \frac{f''(x=x_0)\,(x-x_0)}{2!} + \dots$$

$$= f(x_0) + \frac{f'(x_0)\,(x-x_0)}{1!} + \dots$$

$$= f_0 + f'_0\,\frac{(x-x_0)}{1!} + f''_0\,\frac{(x-x_0)^2}{2!} + \dots$$

x_0 information: $f(x_0) \leftarrow f' \leftarrow f'' \dots$ x_0 … nth



x_0 = [illegible]

$\rightarrow x_0 = 0 \rightarrow$ Mclaurin

$y = f(x)$

$\approx f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)(x - x_0)^2}{2!}$

(Taylor Maclaurin)

$x_0 = 0$

$y = f_0 + f'_0 x + \frac{f''_0 x^2}{2!} + f'''_0 + \ldots$ order 5

$= a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n + \ldots$

$a_0 = f(x=0)$

$a_1 = \frac{f'(x=0)}{1!}, \quad a_2 = \frac{f''_0}{2!}$

$f''_0 = f''(x=0) = \left.\frac{d^2 f}{dx^2}\right|_{x=0}$

constant

(*) $y = a_0 + a_1 x + a_2 x^2 + \dots a_n x^n + \dots$

Pendekatan nilai Var y tertentu berupa pd var. bebas nya x

Taylor → mclaurin (*) Power series / Pangkat

(1) $y^{(x)} = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots a_n x^n + \dots$

$y'(x) = \cancel{x + 2x + 3x^2 + \dots x}$

(2) $y'(x) = a_1 + 2a_2 x + 3a_3 x^2 + \dots + n a_n x^{n-1} + \dots$

PD orde 1

Contoh

$$y' + 2y = 10$$

V_0 10 Volt.

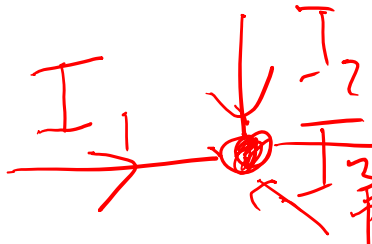
Pers. Kirchuf

$I_{masuk} = I_{keluar}$

$I_c = I$

$A = C$

$B = D$

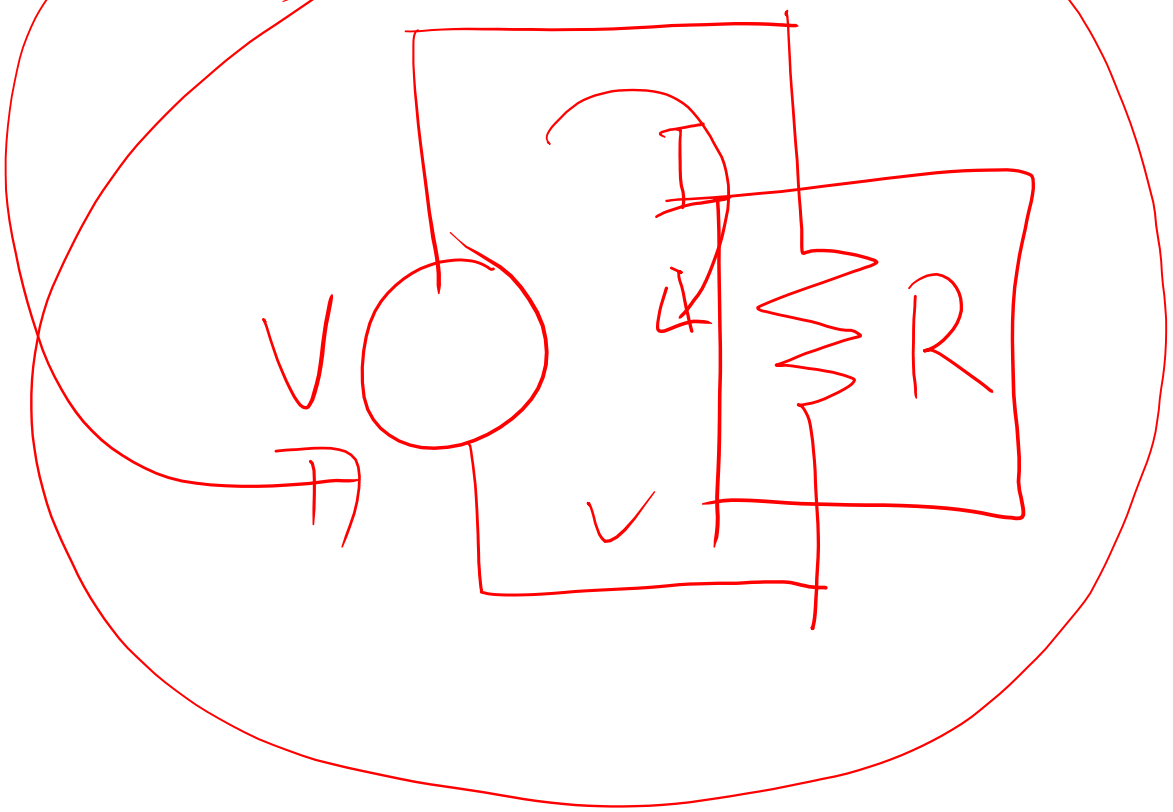


$I_p \neq I_1 + I_2$

I_3

$I_1 + I_2 = I_3$

$y' + 5y = 10$



$V = IR$

$10 = RI$

$I = y$

$RI = 10$

$y = 10$

Sebuah PD, contoh

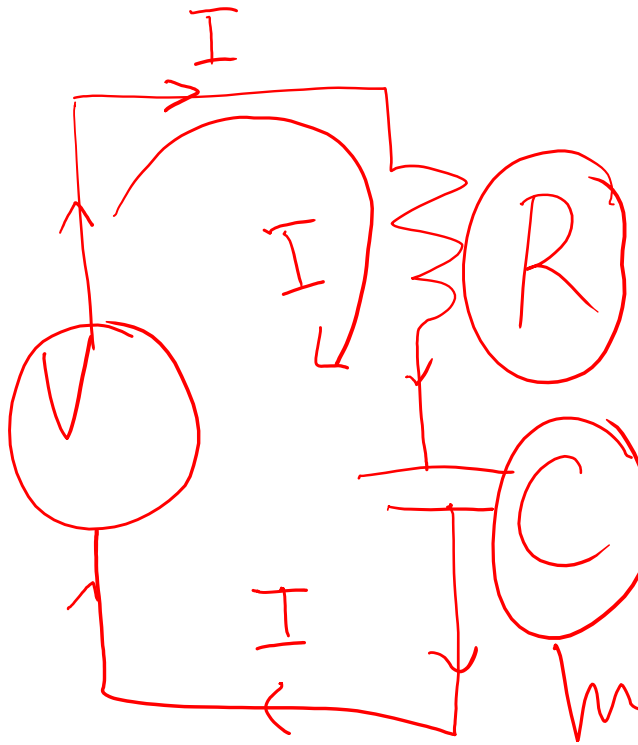
$$y'' + p(x)y' + q(x)y = 0$$

Penyelesaian PD di atas dilakukan dengan menggunakan Deret Pangkat

$$(2)\quad \sum_{m=0}^{\infty} a_m x^m = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots.$$

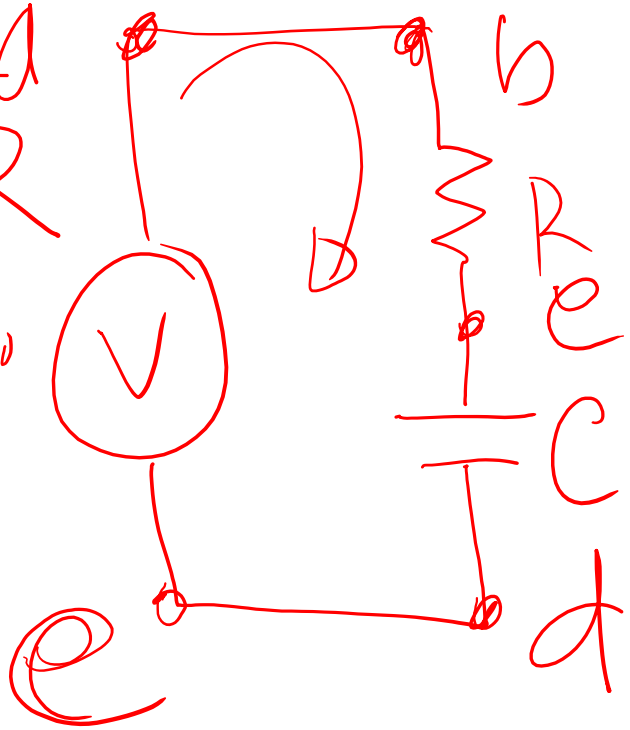
$$(3)\quad y' = a_1 + 2a_2 x + 3a_3 x^2 + \cdots = \sum_{m=1}^{\infty} m a_m x^{m-1}$$

$$(5)\quad y'' = 2a_2 + 3\cdot 2a_3 x + 4\cdot 3a_4 x^2 + \cdots = \sum_{m=2}^{\infty} m(m-1) a_m x^{m-2}$$


I
I
R
V
C
I

$$\frac{dQ}{dt} + \frac{1}{RC}Q = \frac{V_0}{R}$$

$$R\frac{dQ}{dt} + \frac{1}{C}Q = V_0$$



$$V_0 = IR + \frac{Q}{C}$$

$$V = \frac{dQ}{dt}$$

Hukum Ohm

$$\sum V = IR$$

$$V_{ac} = V_{ab} + V_{bc} + V_{cd} + V_{de}$$

$$= IR + \frac{Q}{C}$$

$$V_{hukum} = IR + \frac{Q}{C}$$

$$\frac{dq}{dt} + \frac{1}{RC} q = \frac{1}{R} V_0$$

$$\frac{dy}{dt} + \frac{1}{RC} y = \frac{V_0}{R}$$

$$y'(x) + 5y(x) = 10$$

$$y' + 5y = 10$$

$$y = a_0 + a_1$$

Rumus $x = ?$

$$\frac{dy}{dx} + 5y = 10 \quad x = ?$$

(a) $\frac{dy}{dx} + 5y = 10$

(1) $y = a_0 + a_1 x + a_2 x^2 + \bigcirc + a_n x^n +$

(2) $y' = a_1 + 2a_2 x + 3a_3 x^2 + \ldots$

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Beberapa sifat dari Deret Pangkat

1. Turunan / Diferensiasi

$$y(x) = \sum_{m=0}^{\infty} a_m (x - x_0)^m$$

$$y'(x) = \sum_{m=1}^{\infty} m a_m (x - x_0)^{m-1} \qquad (|x - x_0| < R).$$

y”(x) = ...

2. Penjumlahan Deret Pangkat

$$\sum_{m=0}^{\infty} a_m (x - x_0)^m \quad \text{and} \quad \sum_{m=0}^{\infty} b_m (x - x_0)^m$$

$$\sum_{m=0}^{\infty} (a_m + b_m)(x - x_0)^m$$

3. Perkalian

$$a_0 b_0 + (a_0 b_1 + a_1 b_0)(x - x_0) + (a_0 b_2 + a_1 b_1 + a_2 b_0)(x - x_0)^2 + \cdots$$

$$= \sum_{m=0}^{\infty} (a_0 b_m + a_1 b_{m-1} + \cdots + a_m b_0)(x - x_0)^m$$

(0) $y' + 5y = 10$

(1) $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$

(2) $y' = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \dots$

$(a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \dots)$

$+ 5(a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots) = 10$

$(a_1 + 5a_0) x^0 + (2a_2 + 5a_1) x^1 + (\dots\dots) = 10 \cdot x^0$

$a_1 + 5a_0 = 10$; $2a_2 + 5a_1 = 0$

$n(3a_3 + 5a_2) = 0$

dst.

$a_1 + 5a_0 = 10$

$2a_2 + 5a_1 = 0$

$3a_3 + 5a_2 = 0$

$4a_4 + 5a_3 = 0$

$5a_5 + 5a_4 = 0$

⋮

usw.

$\rightarrow a_1 = 10 - 5a_0$

$2a_2 = -5(10 - 5a_0)$

$= -5 \cdot 10 + 5^2 a_0$

$a_2 = \frac{1}{2}(-5 \cdot 10 + 5^2 a_0)$

$a_3 = \frac{1}{3 \cdot 2}(+5 \cdot 10 - 5^2 a_0)$

usw.

$$\frac{dy}{dx} + 5y = 0$$

$$y = a_0 + a_1 x + \dots$$

$$y' = a_1 + 2a_2 x + 3a_3 x^2 + \dots$$

$$a_1 + 5a_0 = 0$$

$$a_1 = -5a_0$$

$$2a_2 + 5a_1 = 0$$

$$a_2 = -\frac{5}{2} a_1 = +\frac{5^2 a_0}{2}$$

$$a_3 = -\frac{5^3 a_0}{3 \cdot 2}$$

$$a_4 = \frac{5^4 a_0}{4!}$$

$$\vdots$$

$$a_n = \frac{5^n a_0}{n!}$$

n quy nạp

$$a_1 = -5a_0$$

$$a_2 = \frac{5^2 a_0}{2!}$$

$$a_3 = -\frac{5^3 a_0}{3!}$$

$$a_n = \frac{5^n a_0}{n!} \qquad y = a_0 \left(1 - (5x) + \frac{(5x)^2}{2!} - \frac{(5x)^3}{3!} + \dots\right)$$

$$a_n = (-1)^n \frac{5^n a_0}{n!} \qquad y = a_0 e^{-5x}$$

$$e^x = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!}$$

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$$

$$= a_0 - 5a_0 x + \frac{5^2 a_0 x^2}{2!} - \frac{5^3 a_0 x^3}{3!} + \dots$$

$$= \left(1 - 5x + \frac{5^2}{2!}x^2 - \frac{5^3}{3!}x^3 + \frac{5^4}{4!}x^4 - \frac{5^5}{5!}x^5 \dots\right) a_0$$

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soal

Selesaikan PD berikut dengan deret Pangkat

1. $y' + 2y = 0$
2. $y'' + 3y' - 2y = 0$
3. $y'' + 3y' - 2y = 1$
4. $y'' + 3y' - 2y = x$
5. $y'' + 3y' - 2y = \cos x$

(1) $y = a_0 + a_1 x + \cdots$

(2) $y' = a_1 + 2a_2 x + 3a_3 x^2 \cdots$

(3) $y'' = 2a_2 + 3 \cdot 2 a_3 x + \cdots$

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Catat semua perkuliahan secara sinkron

Terimakasih

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