FUNGSI GAMMA DAN BETA

FUNGSI GAMMA

1. $\Gamma(n) = \int_0^\infty x^{n-1} e^{-x}\, dx, n > 0$

$$\Gamma(n+1) = n\Gamma(n)$$

$$\Gamma(1) = 1, \quad \Gamma(2) = 1$$

$$\Gamma(n+1) = n!, \; n > 0$$

2. $\Gamma(n) = 2\int_0^\infty y^{2n-1} e^{-y^2}\, dy; \; \Gamma\left(\tfrac{1}{2}\right) = +\sqrt{\pi}$

Nilai Γ(n), untuk 1≤n≤2, diketahui dari Tabel Fungsi Gamma. (n bilangan pecahan).

Contoh:

1. $\Gamma(6) = 5! = 5.4.3.2.1 = 120$
2. $\Gamma(2{,}40) = \Gamma(1{,}40 + 1) = 1{,}40\ \Gamma(1{,}40)$

   $= 1{,}40\ (0{,}887264)$

   $= 1{,}24217$

3. $I = \int_0^\infty x^5 e^{-x} dx = \int_0^\infty x^{6-1} e^{-x} dx = \Gamma(6) = 5! = 120$

4. $I = \int_0^\infty x^4 e^{-2x}\, dx = \int_0^\infty \left(\tfrac{1}{2} y\right)^4 e^{-y}\, \tfrac{1}{2}\, dy = \tfrac{1}{32} \int_0^\infty y^{5-1} e^{-y}\, dy$

$= \frac{1}{32}\Gamma(5) = \frac{1}{32} 4! = \frac{3}{4}$

Misal: 2x = y

x = ½ y

Untuk: x=0 →y=0

x =∞→y=∞

Dari x= ½ y → dx = ½ dy

Fungsi Beta

1. $$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx, \quad m>0,\ n>0$$

2. $$B(m,n) = 2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2n-1}\theta\,d\theta$$

3. $$B(m,n) = \frac{1}{a^{m+n-1}}\int_0^a y^{m-1}(a-y)^{n-1}\,dy$$

4. $$B(m,n) = \int_0^\infty \frac{y^{m-1}}{(1+y)^{m+n}}\,dy = \int_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}}\,dy$$

Hubungan fungsi Beta dan Gamma: $B(m, n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

1. $$\int_0^1 x^{m-1}(1-x)^{n-1}\,dx = B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

2. $$\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2n-1}\theta\, d\theta = \frac{1}{2}B(m, n) = \frac{1}{2}\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

3. $$\int_0^a y^{m-1}(a-y)^{n-1}\,dy = a^{m+n-1}B(m, n) = a^{m+n-1}\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

4. $$\int_0^\infty \frac{y^{m-1}}{(1+y)^{m+n}}\,dy = \int_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}}\,dy = B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

Rumus: $\Gamma(p)\Gamma(1-p) = \dfrac{\pi}{\sin p\pi}$

Contoh:

$$1.\ I = \int_0^1 x^5 (1-x)^3\, dx = \int_0^1 x^{6-1}(1-x)^{4-1}\, dx = B(6,4) = \frac{\Gamma(6)\Gamma(4)}{\Gamma(6+4)}$$

$$= \frac{5!\ 3!}{9!} = \frac{5!3.2.1}{9.8.7.6.5!} = \frac{1}{504}$$

$$2.\ I = \int_0^{\frac{\pi}{2}} \sin^5 x\ \cos^3 x\, dx = \frac{1}{2} B(m,n) = \frac{1}{2} B(3,2) = \frac{1}{2}\frac{\Gamma(3)\Gamma(2)}{\Gamma(3+2)}$$

$2m-1=5 \rightarrow m=3$

$2n-1=3 \rightarrow n=2$

$$= \frac{1}{2}\frac{2!\ 1!}{4!}$$

3. $\int_0^{\pi} \sin^5 x\,dx = 2\int_0^{\pi/2} \sin^5 x \cos^0 x\,dx = 2.\frac{1}{2}B(3,\frac{1}{2}) = \frac{\Gamma(3)\Gamma(\frac{1}{2})}{\Gamma(3+\frac{1}{2})}$

2m-1=5→m=3

2n-1=0→n= ½

$$= \frac{2!\sqrt{\pi}}{\frac{5}{2}\cdot\frac{3}{2}\frac{1}{2}\sqrt{\pi}} = \frac{16}{15}$$

4. $I = \int_0^3 x^2(3-x)^4\,dx = a^{m+n-1}B(m,n) = 3^{3+5-1}B(3,5) = 3^7\,\frac{\Gamma(3)\Gamma(5)}{\Gamma(3+5)}$

a=3; m-1=2; n-1=4

m=3 n=5

$$= 3^7\,\frac{2!4!}{7!} = \frac{3^6}{35}$$

Soal Latihan:

1. $I = \int_0^\infty x^4 e^{-x}\, dx$

2. $I = \int_0^\infty t^6 e^{-3t}\, dt$

3. $\Gamma(-3{,}5)$

4. $\Gamma(2{,}6)$

5. $I = \int_0^1 x^2 (1-x)^3\, dx$

6. $I = \int_0^{\frac{\pi}{2}} \cos^5 x \sin^2 x\, dx$

7. $I = \int_0^2 x^{5/2} (2-x)^{1/2}\, dx$

8. $I = \int_0^{\pi/8} \cos^3 4x\, dx$