ADVANCED ELECTRICAL CIRCUIT
BETI 1333
TRANSFER FUNCTION AND SERIES RESONANCE

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LESSON OUTCOMES

At the end of this chapter, students are able:

- to describe transfer function and determine characteristics of series resonance
- to illustrate frequency response of transfer function

SUBTOPICS

- Transfer Function
- Series Resonance

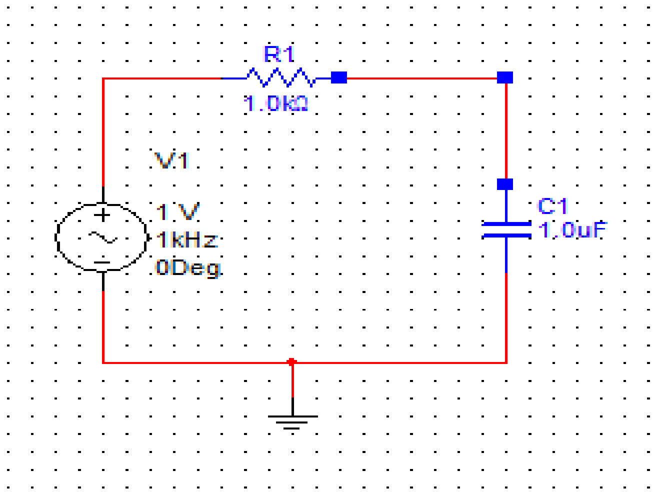
INTRODUCTION

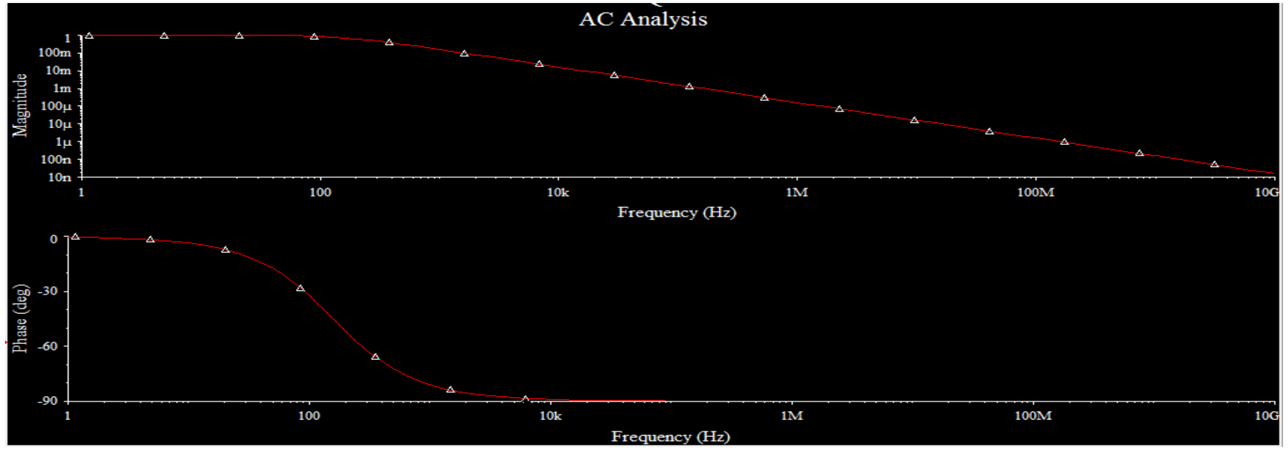
What?

1. Variation in a circuit's behaviour

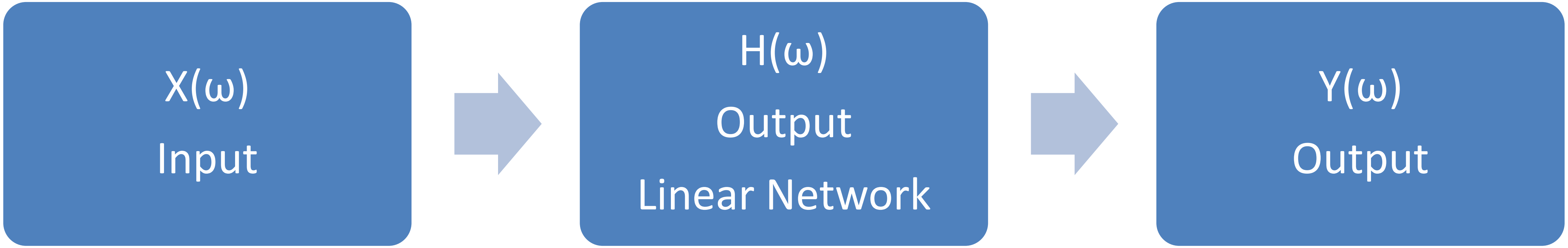
2. Change in signal frequency

3. Variation of the gain and phase with frequency





TRANSFER FUNCTION



$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = |H(\omega)| \angle \phi$$

TRANSFER FUNCTION

Voltage gain

$$H(\omega) = \frac{Vo(\omega)}{Vi(\omega)}$$

Current gain

$$H(\omega) = \frac{Io(\omega)}{Ii(\omega)}$$

Transfer impedance

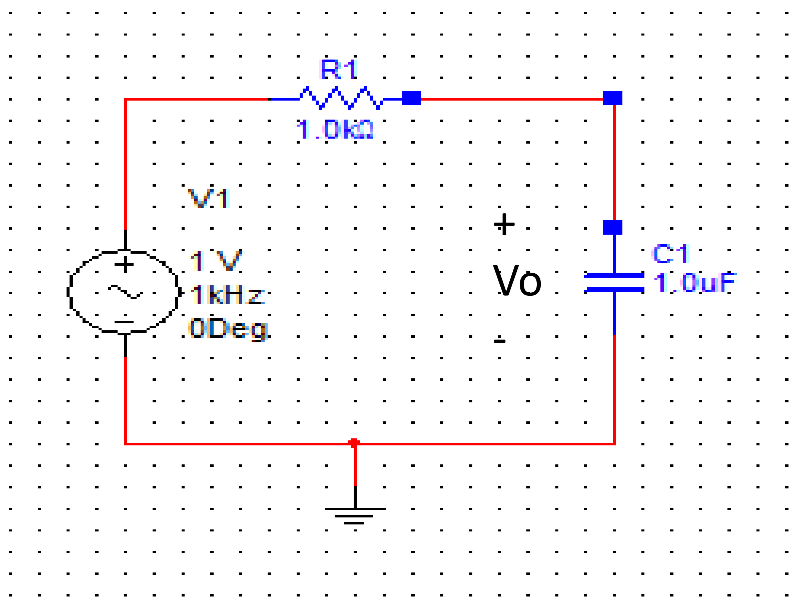
$$H(\omega) = \frac{Vo(\omega)}{Ii(\omega)}$$

Transfer Admittance

$$H(\omega) = \frac{Io(\omega)}{Vi(\omega)}$$

EXAMPLE 1

Find the transfer function V_o/V_i and frequency response of circuit below.



SOLUTION 1

STEP 1 :

The transfer function:

$$H(\omega) = \frac{V_o}{V_i} = \frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} = \frac{1}{1+j\omega RC}$$

$$H(\omega) = \frac{1}{1+j\omega 0.001}$$

STEP 2 :

The magnitude:

$$|H(\omega)| = \frac{1}{\sqrt{1+(\frac{\omega}{\omega o})^2}}$$

$$|H(\omega)| = \frac{1}{\sqrt{1+(\frac{\omega}{1000})^2}}$$

STEP 3 :

The phase:

$$\phi = -\tan^{-1}\frac{\omega}{\omega_o}$$

$$\phi = -\tan^{-1}\frac{\omega}{1000}$$

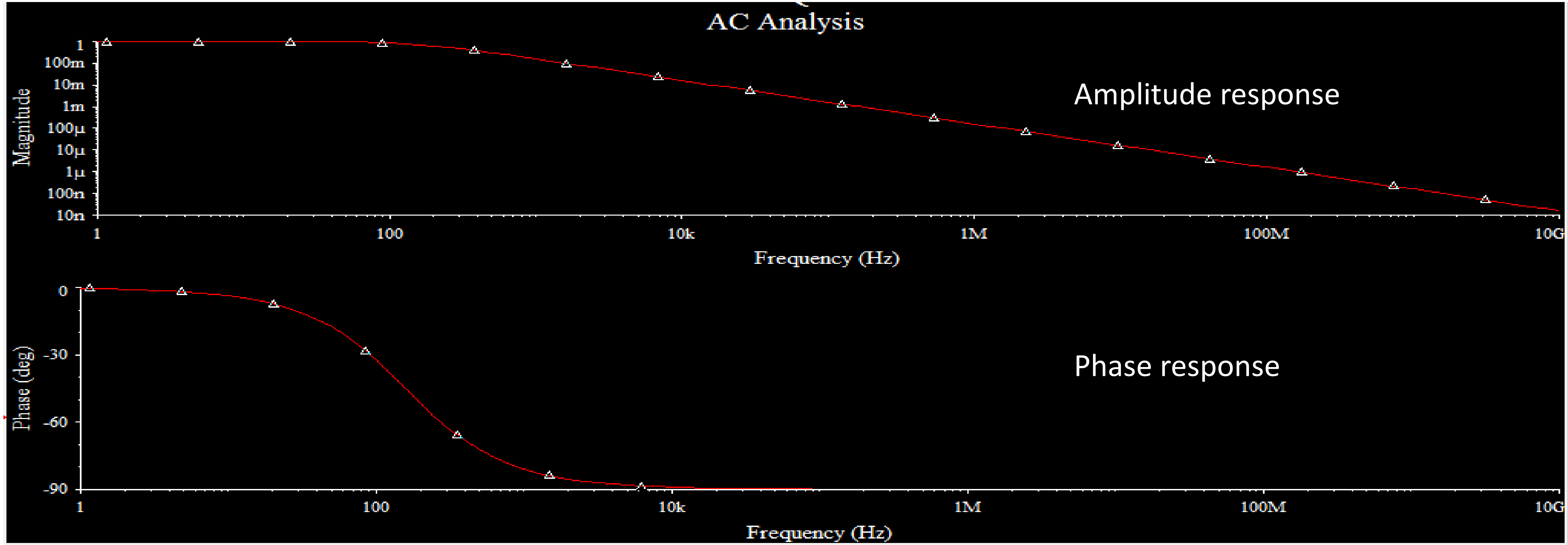
$j\omega RC = j\omega(1k\Omega)(1\mu F) = j\omega\ 0.001$

Angular frequency:

$$\omega_o = \frac{1}{RC} = \frac{1}{1000\Omega(1\mu F)} = 1000$$

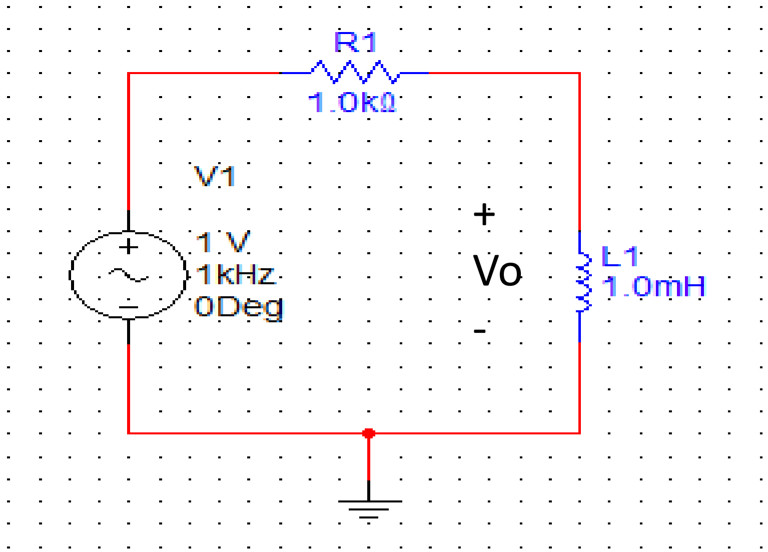
SOLUTION 1

Frequency response for the RC circuit:



EXAMPLE 2

Find the transfer function V_o/V_i and frequency response of circuit below.



SOLUTION 2

STEP 1 :

The transfer function:

$$H(\omega) = \frac{V_o}{V_i} = \frac{j\omega L}{R+j\omega L} = \frac{1}{1+\frac{R}{j\omega L}}$$

$$H(\omega) = \frac{1}{1+\frac{1000}{j\omega 0.001}}$$

STEP 2 :

The magnitude:

$$|H(\omega)| = \frac{1}{\sqrt{1+(\frac{\omega}{\omega o})^2}}$$

$$|H(\omega)| = \frac{1}{\sqrt{1+(\frac{\omega}{1000})^2}}$$

STEP 3 :

The phase:

$$\phi = \angle 90^\circ - \tan^{-1}\frac{\omega}{\omega_o}$$

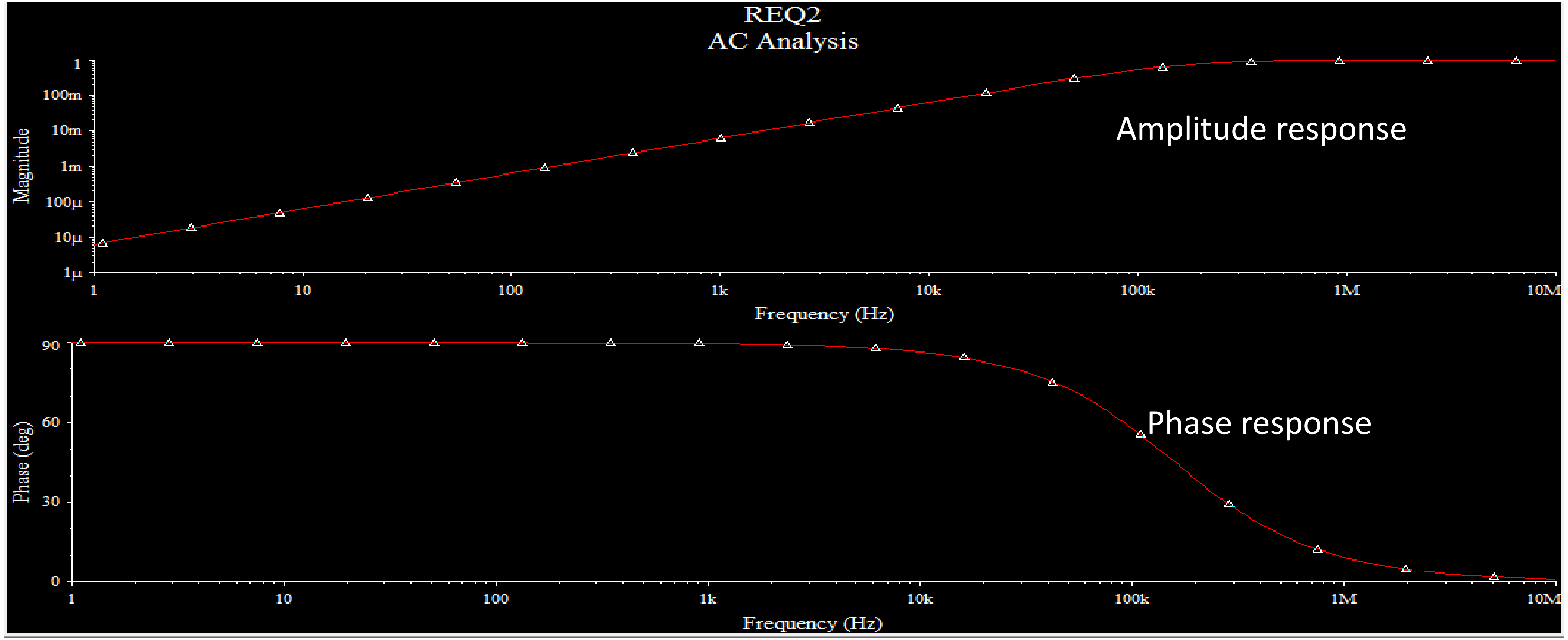
$$\phi = \angle 90^\circ - \tan^{-1}\frac{\omega}{1000000}$$

$$\frac{R}{j\omega L} = \frac{1000}{j\omega 0.001}$$

$$\omega_0 = \frac{R}{L} = \frac{1000}{0.001} = 1000000$$

SOLUTION 2

Frequency response for the RL circuit:



SERIES RESONANCE

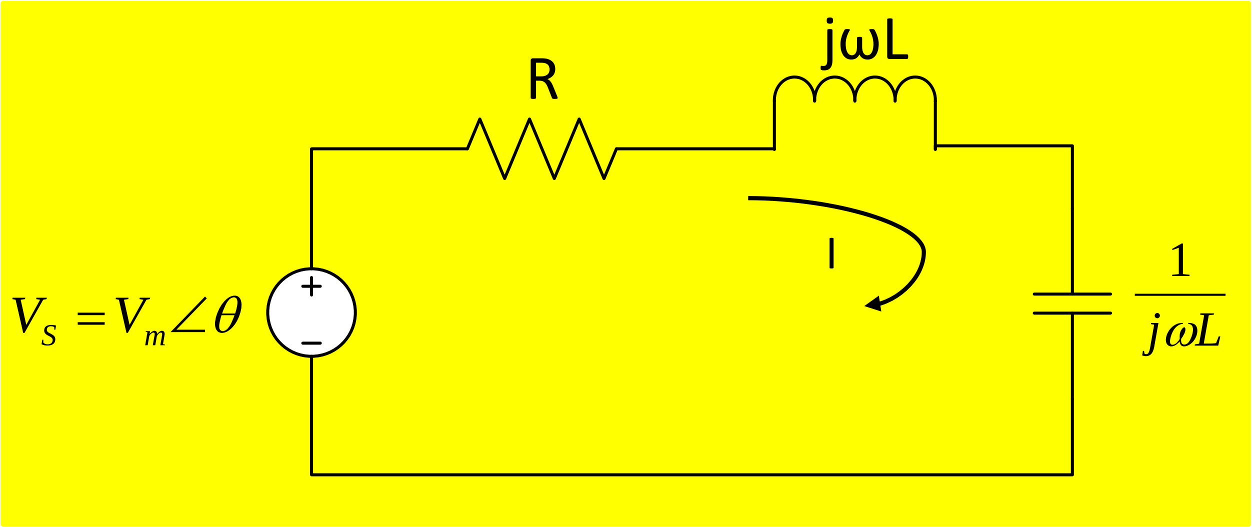
1. Capacitive & inductive reactance must equal magnitude in RLC circuit

+

2. Purely resistive impedance

→ SERIES RESONANCE

SERIES RESONANCE



$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \, rad/s$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \, Hz$$

Resonance frequency

SERIES RESONANCE

Characteristics

1. $Z = R$

2. V_s & I in phase ; $\therefore \cos\theta = 1$

3. $H(\omega) = Z(\omega)$ is minimum

3. L & C > V_s

SERIES RESONANCE

Bandwidth (B)

The current's of frequency response is

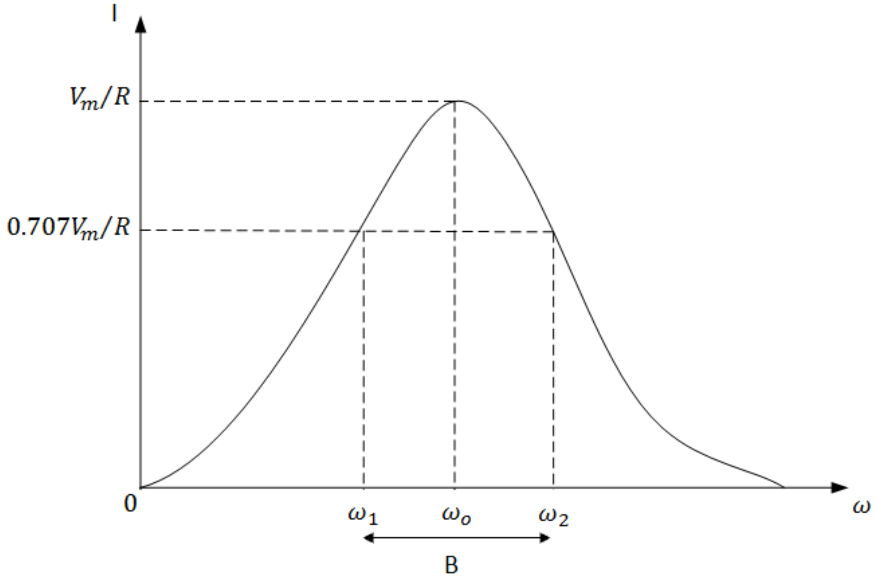
$$I = |I| = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

From RLC circuit, the average power is :

$$P(\omega) = \frac{1}{2} I^2 R$$

So, the highest power dissipated at resonance is :

$$P(\omega_o) = \frac{1}{2}\frac{V_m^2}{R}$$



SERIES RESONANCE

- ω_1 & ω_2 = Half–power frequencies at power dissipated is half the maximum value :

$$P(\omega_1) = P(\omega_2) = \frac{1}{2}\frac{(V_m/\sqrt{2})^2}{R} = \frac{V_m^2}{4R}$$

- with set up Z = $\sqrt{2}$R, then the half-power frequencies :

$$\omega_1 = -\frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}}$$

$$\omega_o = \sqrt{\omega_1 \omega_2}$$

∴ Bandwidth : $B = \omega_2 - \omega_1$

SERIES RESONANCE

Quality factor, Q:

$$Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

Bandwidth, B:

$$B = \frac{R}{L} = \frac{\omega_o}{Q} = \omega_o^2 CR$$

For Q $\geq$ 10:

$$\omega_1 = \omega_0 - \frac{B}{2}$$

$$\omega_1 = \omega_0 - \frac{B}{2}$$

EXAMPLE 3

Given circuit in Figure 1:

a) Calculate the resonant frequency

b) Find the quality factor and bandwidth

c) Determine half power frequencies

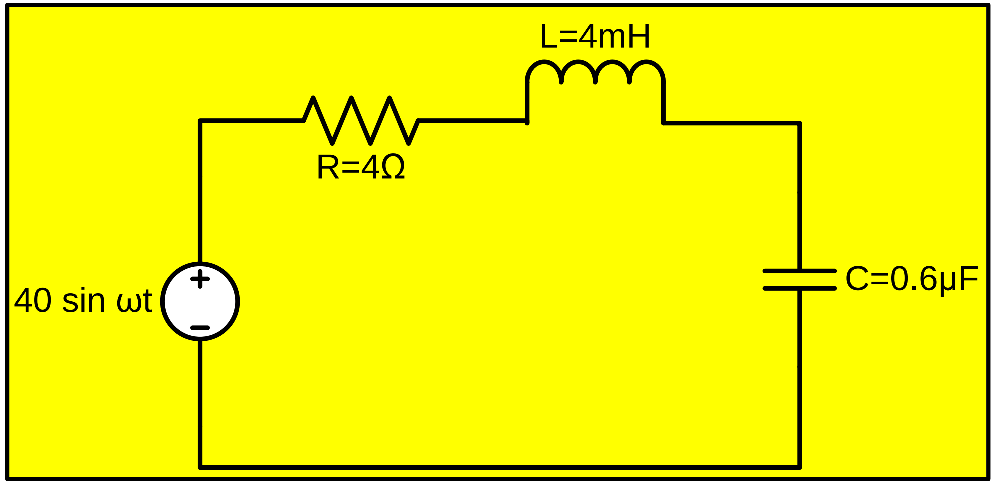


Figure 1

SOLUTION 3

a) Resonant frequency:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\therefore \omega_0 = 20krad/s$$

b) Bandwidth:

$$B = \frac{R}{L}$$

$$\therefore B = 1000rad/s$$

Quality Factor:

$$Q = \frac{\omega_0}{B}$$

$$\therefore Q = 20$$

SOLUTION 3

c) Half-power frequencies:

$$\omega_1 = \omega_0 - \frac{B}{2}$$

$$\therefore \omega_1 = 19.5 krad/s$$

$$\omega_1 = \omega_0 - \frac{B}{2}$$

$$\therefore \omega_2 = 20.5 krad/s$$

SELF REVIEW QUESTIONS

1. The transfer function is a ratio of a phasor output to a phasor input.

   TRUE FALSE

2. The impedance for an inductor is ________.

3. Given R = 5 Ω and L = 10 H are arranged in series. What is the angular frequency for this RL circuit?

   a) 0.5 s b) 1 s

   c) 2 s d) 10 s

4. Identify the difference between the half-power frequencies:

   a) quality factor

   b) resonant frequency

   c) bandwidth

   d) cutoff frequency

5. Give one (1) of the characteristics in series resonance.

   Answer: ______________

ANSWERS

1. TRUE
2. *$j\omega L$*
3. a
4. c
5. i) Z = R

   ii) Vs & I in phase ; $\cos\theta = 1$

   iii) $H(\omega) = Z(\omega)$ is minimum

   iv) inductor (L) & capacitor C > Vs

   **choose any of the answer above