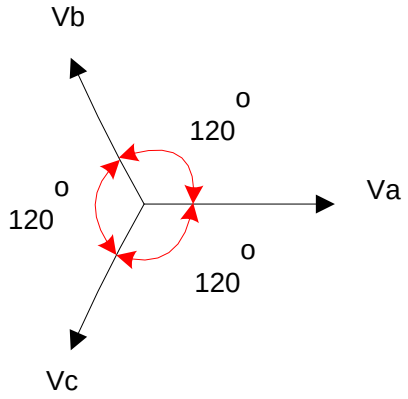
UNBALANCED THREE-PHASE POWER FLOW

Definition of balanced system



$$|V_a| = |V_b| = |V_c|$$

If $|V_a| = |V_b| = |V_c|$ and each other separated by an angle of 120^0

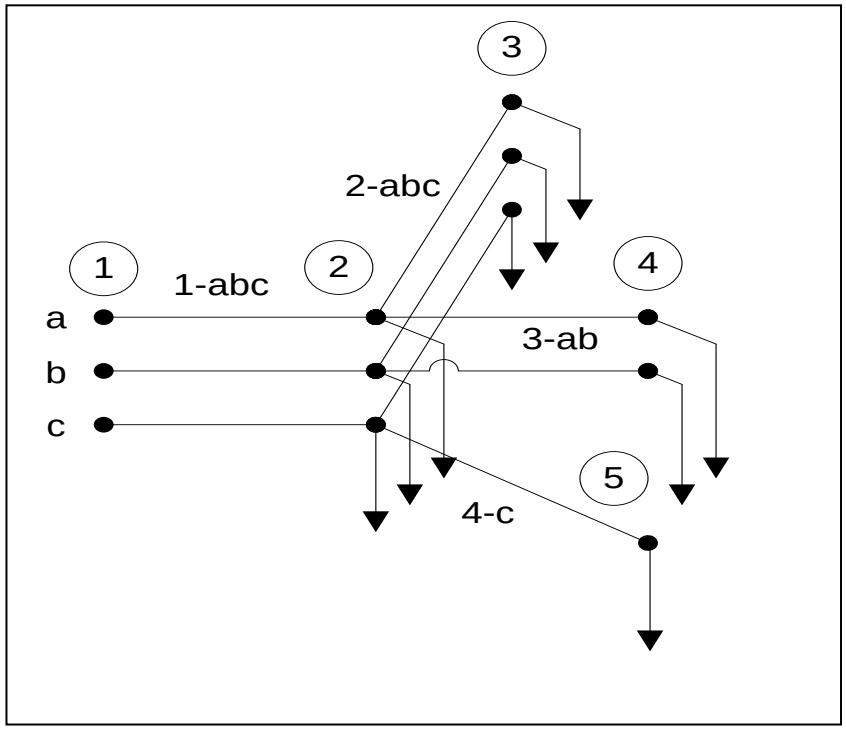
That is called balanced system. Otherwise, it is called unbalanced system.

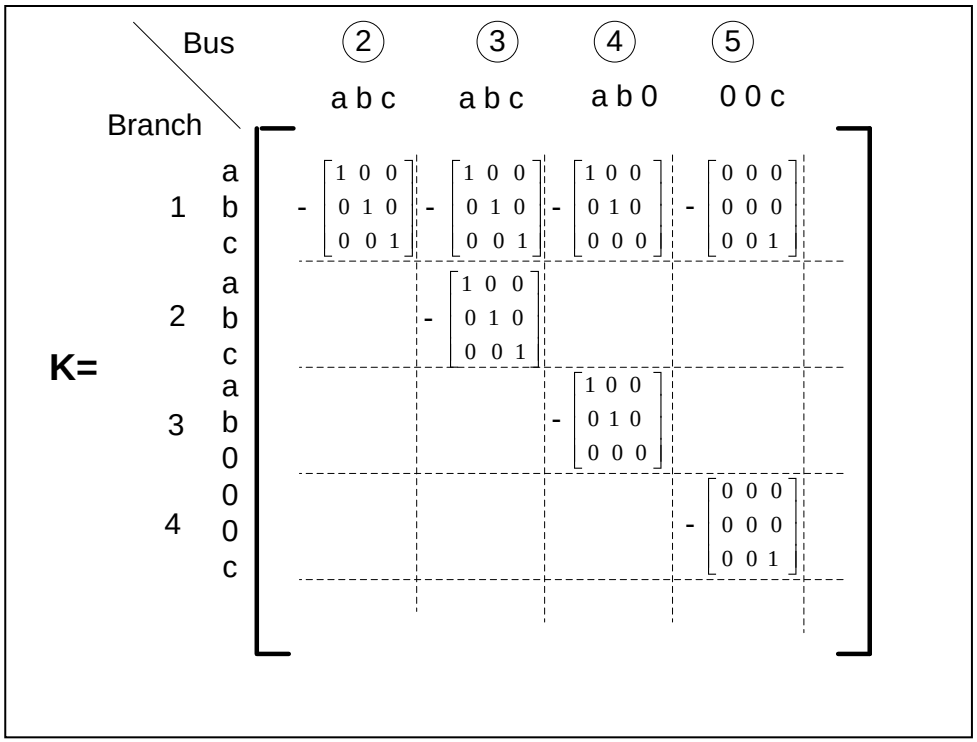
Balanced system ➔ Single phase load flow

Unbalanced system ➔ Three phase load flow

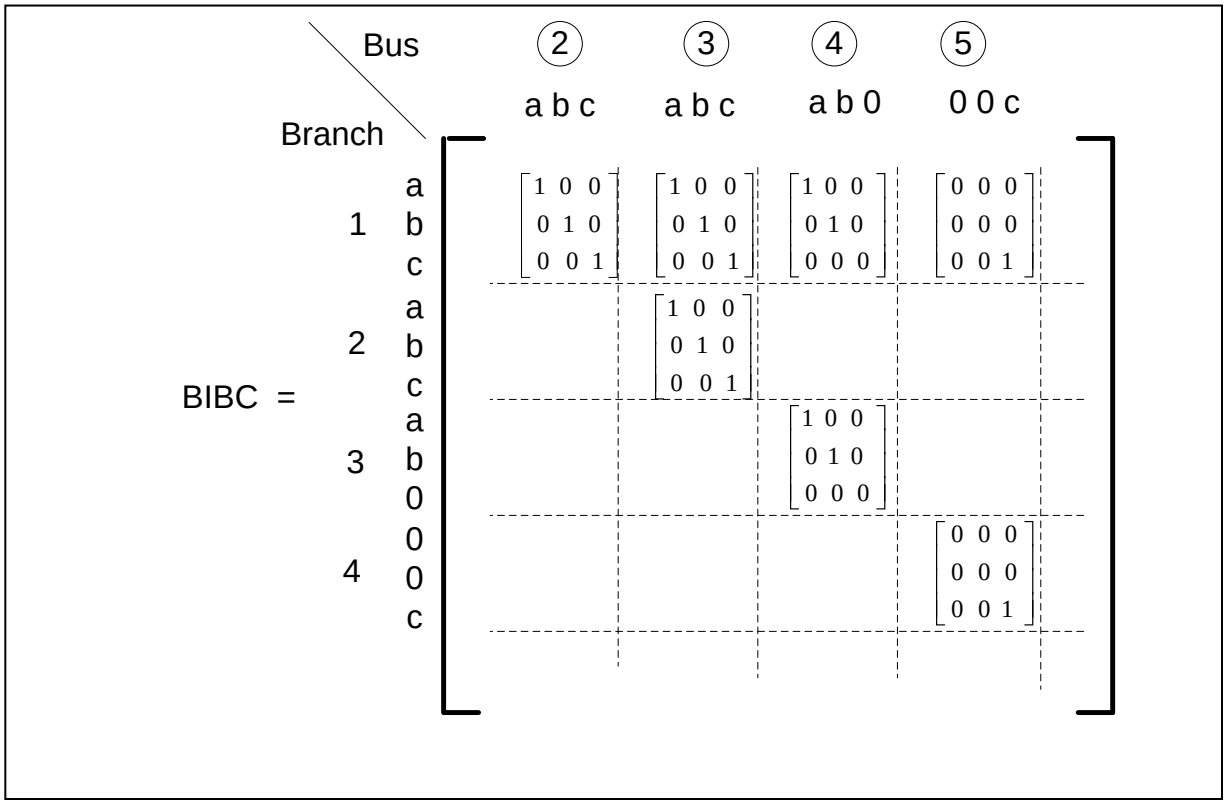
Unbalanced three phase power flow based on topology network

1. Building BIBC-Matrix using K-Matrix

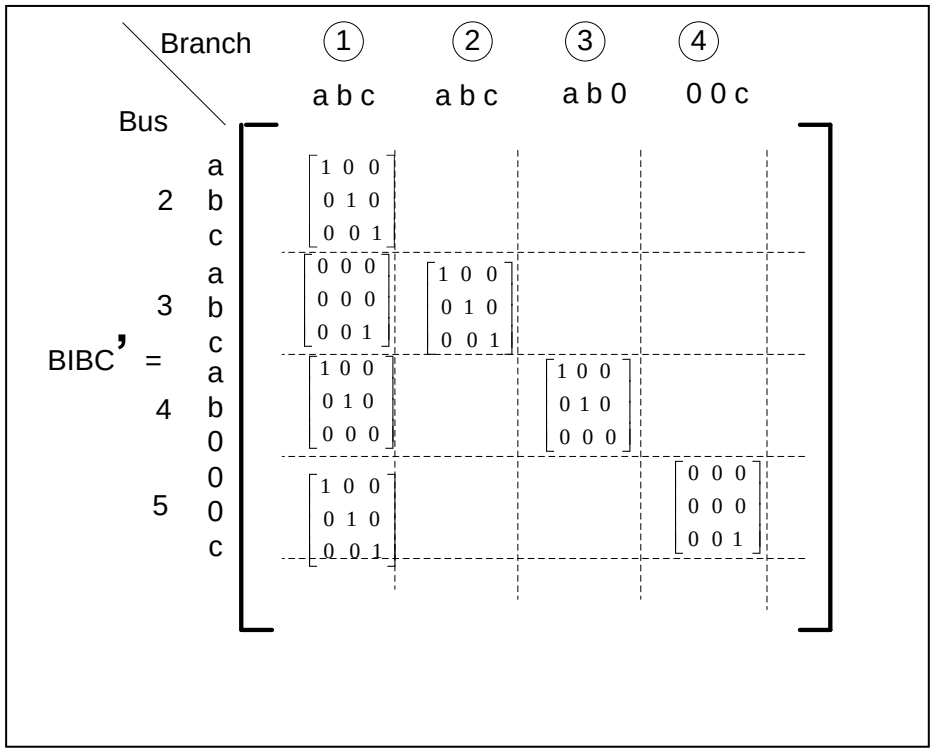


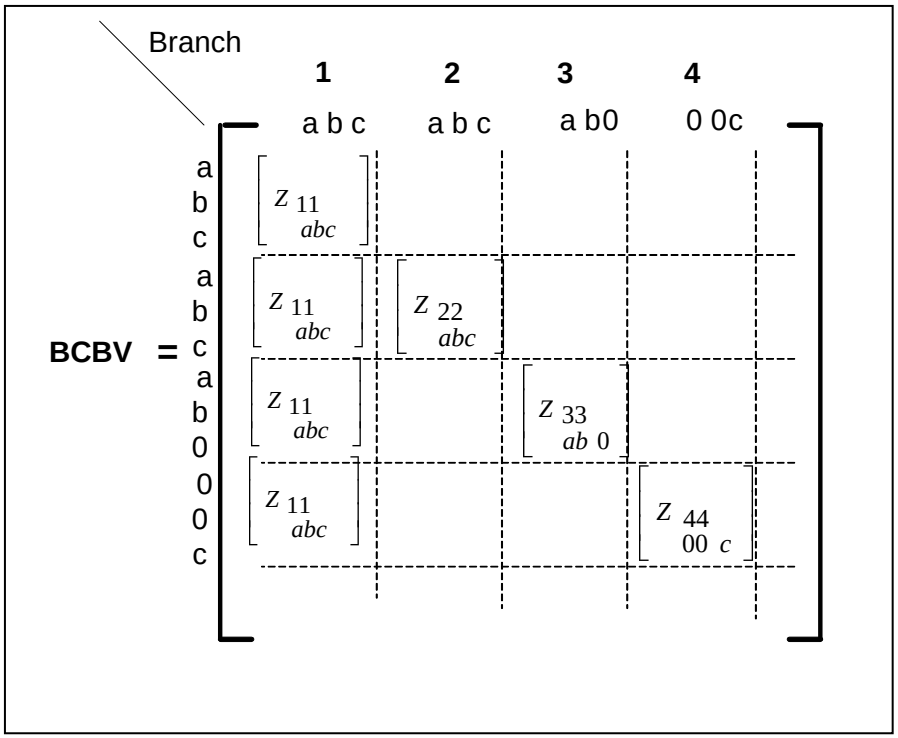


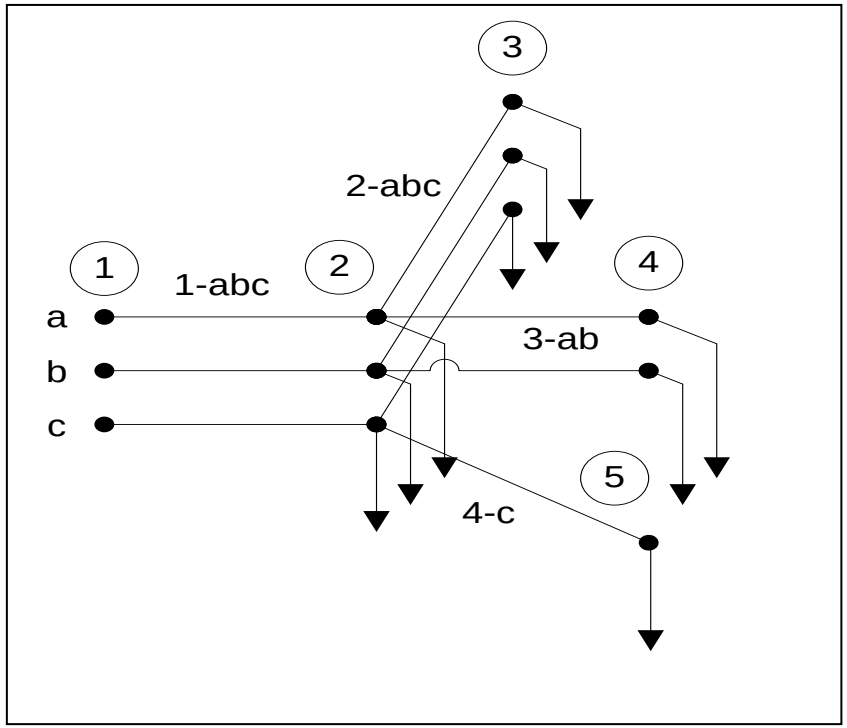
BIBC Matrix = - [K -Matrix]

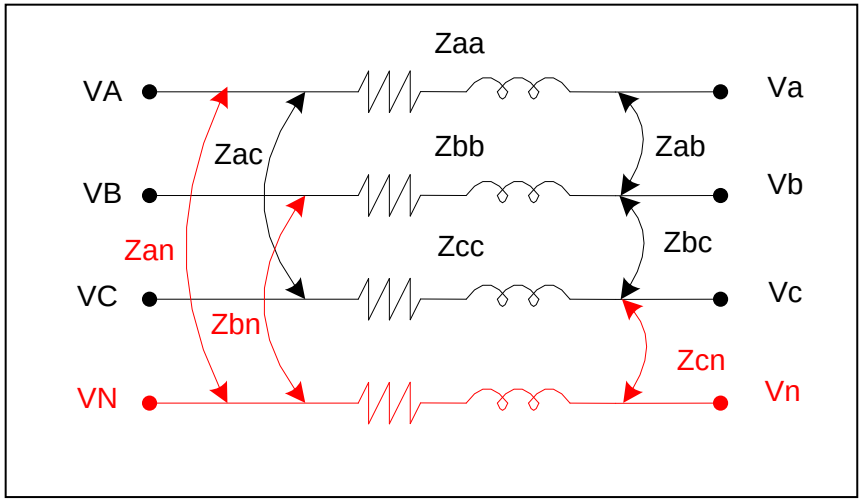

Bus
2
3
4
5
a b c
a b c
a b 0
0 0 c
Branch
BIBC =
1 a
b
c
2 a
b
c
3 a
b
0
4 0
0
c
1 0 0
0 1 0
0 0 1
1 0 0
0 1 0
0 0 1
1 0 0
0 1 0
0 0 0
0 0 0
0 0 0
0 0 1
1 0 0
0 1 0
0 0 1
1 0 0
0 1 0
0 0 0
0 0 0
0 0 0
0 0 1

2. Building BCBV Matrix using transpose of BIBC matrix





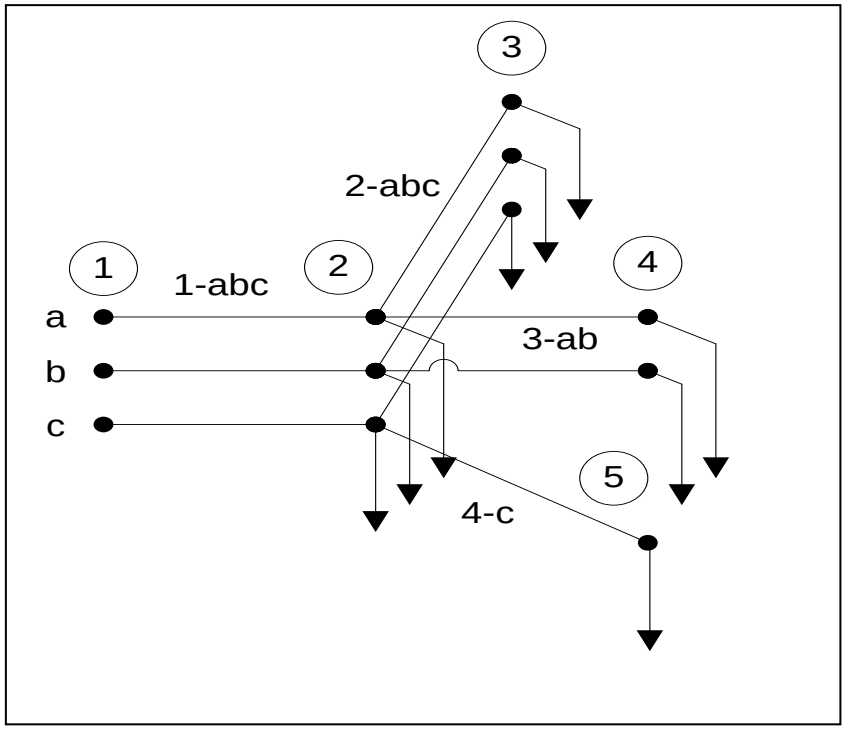




$$[Z_{abc}] = \begin{bmatrix} Z_{aa-n} & Z_{ab-n} & Z_{ac-n} \\ Z_{ba-n} & Z_{bb-n} & Z_{bc-n} \\ Z_{ca-n} & Z_{cb-n} & Z_{cc-n} \end{bmatrix}$$

$$[Z11_{abc}] = \begin{bmatrix} Z_{aa-n} & Z_{ab-n} & Z_{ac-n} \\ Z_{ba-n} & Z_{bb-n} & Z_{bc-n} \\ Z_{ca-n} & Z_{cb-n} & Z_{cc-n} \end{bmatrix}$$

$$[Z22_{abc}] = \begin{bmatrix} Z_{aa-n} & Z_{ab-n} & Z_{ac-n} \\ Z_{ba-n} & Z_{bb-n} & Z_{bc-n} \\ Z_{ca-n} & Z_{cb-n} & Z_{cc-n} \end{bmatrix}$$



$$[Z33_{ab0}] = \begin{bmatrix} Z_{aa-n} & Z_{ab-n} & 0 \\ Z_{ba-n} & Z_{bb-n} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[Z44_{00c}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Z_{cc-n} \end{bmatrix}$$

3. Follow the algorithm.

1. Input data.
2. Build ***BIBC*** matrix**.**
3. Build **BCBV** matrix**.**
4. Build **DLF** matrix**.**
5. Iteration k=0.
6. Iteration k=k+1.
7. Solve for three-phase power flow by using

$$I_i^k = \left(\frac{P_i + jQ_i}{V_i^k}\right)^*$$

And

$$[\Delta V^k] = [\boldsymbol{DLF}][I^k]$$

And update voltages.

$$[V^{k+1}] = [Vbus\_no\ load] - [\Delta V^k]$$

8. If $max_i\left(\left|I_i^{k+1}\right| - \left|I_i^k\right|\right) > tolerance$, go to (6)
9. Report and end.

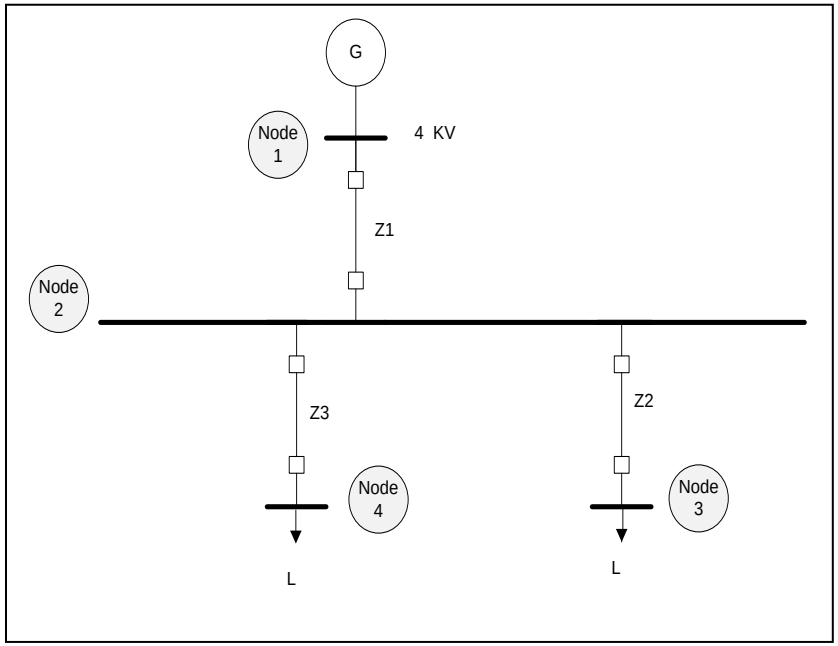
EXAMPLE:

Data:

$$Z = \begin{bmatrix} 0.2 + j0.4 & 0 & 0 \\ 0 & 0.2 + j0.4 & 0 \\ 0 & 0 & 0.2 + j0.4 \end{bmatrix} \Omega$$

Z1=Z2=Z3=Z

$$L = \begin{bmatrix} 0.2\ MW + j0.8\ MVar \\ 0.2\ MW + j0.8\ MVar \\ 0.2\ MW + j0.8\ MVar \end{bmatrix} \begin{matrix} phase\ a \\ phase\ b \\ phase\ c \end{matrix}$$



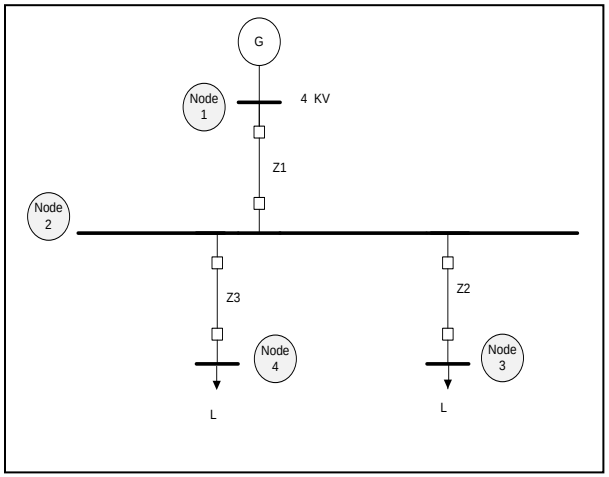
Solution:

1. **Input Data**

   Define Voltage reference;

$$V_a = 4000 < 0^o;\ V_b = 4000 < -120^o;\ V_c = 4000 < 120^o$$

$$Vbus\_no\ load = \begin{bmatrix} 4000 \\ -2000 - 3464.1i \\ -2000 + 3464.1i \\ 4000 \\ -2000 - 3464.1i \\ -2000 + 3464.1i \\ 4000 \\ -2000 - 3464.1i \\ -2000 + 3464.1i \end{bmatrix}$$



2. Build BIBC

$$K = \begin{bmatrix} -1 & -1 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$BIBC = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. Build BCBV

$$BCBV = \begin{bmatrix} Z1 & 0 & 0 \\ Z1 & Z2 & 0 \\ Z1 & 0 & Z3 \end{bmatrix}$$

4. Build DLF

$$DLF = \begin{bmatrix} Z1 & 0 & 0 \\ Z1 & Z2 & 0 \\ Z1 & 0 & Z3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$DLF = \begin{bmatrix} Z1 & Z1 & Z1 \\ Z1 & Z1 + Z2 & Z1 \\ Z1 & Z1 & Z1 + Z3 \end{bmatrix}$$

$$\boxed{DLF = \begin{bmatrix} Z & Z & Z \\ Z & 2Z & Z \\ Z & Z & 2Z \end{bmatrix}}$$

$$DLF = \begin{bmatrix} 0.2 + j0.4 & 0 & 0 & 0.2 + j0.4 & 0 & 0 & 0.2 + j0.4 & 0 & 0 \\ 0 & 0.2 + j0.4 & 0 & Z & 0 & 0.2 + j0.4 & 0 & 0 & 0.2 + j0.4 & 0 \\ 0 & 0 & 0.2 + j0.4 & 0 & 0 & 0.2 + j0.4 & 0 & 0 & 0.2 + j0.4 \\ 0.2 + j0.4 & 0 & 0 & 0.4 + j0.8 & 0 & 0 & 0.2 + j0.4 & 0 & 0 \\ 0 & 0.2 + j0.4 & 0 & 0 & 0.4 + j0.8 & 0 & 0 & 0.2 + j0.4 & 0 \\ 0 & 0 & 0.2 + j0.4 & 0 & 0 & 0.4 + j0.8 & 0 & 0 & 0.2 + j0.4 \\ 0.2 + j0.4 & 0 & 0 & 0.2 + j0.4 & 0 & 0 & 0.4 + j0.8 & 0 & 0 \\ 0 & 0.2 + j0.4 & 0 & 0 & 0.2 + j0.4 & 0 & 0 & 0.4 + j0.8 & 0 \\ 0 & 0 & 0.2 + j0.4 & 0 & 0 & 0.2 + j0.4 & 0 & 0 & 0.4 + j0.8 \end{bmatrix}$$

5. ITERATION

Calculate I

For the 1st iteration V each bus (Vbus) assumed equal to Vref

Iteration, k=1;

$$V_{a=}4000 < 0^o;\ V_{b=}4000 < -120^o; V_{c=}4000 < 120^o$$

$$I_i^k = \left(\frac{P_i + jQ_i}{V_i^k}\right)^*$$

$$I_{4a}^1 = \left(\frac{(0.2 + j0.8)1e6}{4 * 1e3}\right)^* = \left(\frac{(0.2 - j0.8)1e6}{4 * 1e3 < 0^o}\right) = 50.00 - 25.00i\ A$$

$$I_{4a}^1 = I_{3a}^1 = 50.00 - 25.00i\ A$$

$$I_{4b}^1 = \left(\frac{(0.2 + j0.8)1e6}{4 * 1e3 < -120^o}\right)^* = \left(\frac{(0.2 - j0.8)1e6}{4 * 1e3 < -120^o}\right) = -198.21 + 56.70i\ \ A$$

$$I_{4b}^1 = I_{3b}^1 = -198.21 + 56.70i\ \ A$$

$$I_{4c}^{1} = \left(\frac{(0.2 + j0.8)1e6}{4 * 1e3 < 120^{o}}\right)^{*} = \left(\frac{(0.2 - j0.1)1e6}{4 * 1e3 < 120^{o}}\right) = 148.21 \ + \ 143.30i \ A$$

$$I_{4c}^{1} = I_{3c}^{1} = 148.21 \ + \ 143.30i \ A$$

$$I_{2a}^{1} = I_{2b}^{1} = I_{2c}^{1} = 0$$

$$I^{1} = \begin{bmatrix} I_{2a}^{1} \\ I_{2b}^{1} \\ I_{2c}^{1} \\ I_{3a}^{1} \\ I_{3b}^{1} \\ I_{3c}^{1} \\ I_{4a}^{1} \\ I_{4b}^{1} \\ I_{4c}^{1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 50.00 - 25.00i \\ -198.21 \ + \ 56.70i \\ 148.21 \ + \ 143.30i \\ 50.00 - 25.00i \\ -198.21 \ + \ 56.70i \\ 148.21 \ + \ 143.30i \end{bmatrix}$$

Calculate ΔV_1

$$[\Delta V] = [\boldsymbol{DLF}][I]$$

$$[\Delta V_1] \begin{bmatrix} 0.2+j0.4 & 0 & 0 & 0.2+j0.4 & 0 & 0 & 0.2+j0.4 & 0 & 0 \\ 0 & 0.2+j0.4 & 0 & Z & 0 & 0.2+j0.4 & 0 & 0 & 0.2+j0.4 \\ 0 & 0 & 0.2+j0.4 & 0 & 0 & 0.2+j0.4 & 0 & 0 & 0.2+j0.4 \\ 0.2+j0.4 & 0 & 0 & 0.4+j0.8 & 0 & 0 & 0.2+j0.4 & 0 & 0 \\ 0 & 0.2+j0.4 & 0 & 0 & 0.4+j0.8 & 0 & 0 & 0.2+j0.4 & 0 \\ 0 & 0 & 0.2+j0.4 & 0 & 0 & 0.4+j0.8 & 0 & 0 & 0.2+j0.4 \\ 0.2+j0.4 & 0 & 0 & 0.2+j0.4 & 0 & 0 & 0.4+j0.8 & 0 & 0 \\ 0 & 0.2+j0.4 & 0 & 0 & 0.2+j0.4 & 0 & 0 & 0.4+j0.8 & 0 \\ 0 & 0 & 0.2+j0.4 & 0 & 0 & 0.2+j0.4 & 0 & 0 & 0.4+j0.8 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 50.00 - 25.00i \\ -198.21 + 56.70i \\ 148.21 + 143.30i \\ 50.00 - 25.00i \\ -198.21 + 56.70i \\ 148.21 + 143.30i \end{bmatrix}$$

$\Delta V_1 =$

*** 1.0e+002 ***

1.8000 - 0.4000i
-1.2464 - 1.3588i
-0.5536 + 1.7588i
2.7000 - 0.6000i
-1.8696 - 2.0383i
-0.8304 + 2.6383i
2.7000 - 0.6000i
-1.8696 - 2.0383i
-0.8304 + 2.6383i

Iteration, k=2;

$$V_{i=2:4}^{2} = Vbus\_no\ load - \Delta V_1$$

$V_{i=2:3}^{2}$	1.0e+003 *	-	**1.0e+002 ***	=	1.0e+003 *
	4.0000		1.8000 - 0.4000i		3.8200 + 0.0400i
	-2.0000 - 3.4641i		-1.2464 - 1.3588i		-1.8754 - 3.3282i
	-2.0000 + 3.4641i		-0.5536 + 1.7588i		-1.9446 + 3.2882i
	4.0000		2.7000 - 0.6000i		3.7300 + 0.0600i
	-2.0000 - 3.4641i		-1.8696 - 2.0383i		-1.8130 - 3.2603i
	-2.0000 + 3.4641i		-0.8304 + 2.6383i		-1.9170 + 3.2003i
	4.0000		2.7000 - 0.6000i		3.7300 + 0.0600i
	-2.0000 - 3.4641i		-1.8696 - 2.0383i		-1.8130 - 3.2603i
	-2.0000 + 3.4641i		-0.8304 + 2.6383i		-1.9170 + 3.2003i

$$I_{i=2:4}^{2} = \left(\frac{P_{i=2:4} + jQ_{i=2:4}}{V_{i=2:4}^{2}} \right)^{*}$$

Using the same way… the results each iterations can be shown in the table:

I^2	I^3	I^4	I^5	I^6	I^7
1.0e+002 *	1.0e+002 *	1.0e+002 *	1.0e+002 *	1.0e+002 *	1.0e+002 *
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0.5705 - 2.1356i	0.5737 - 2.1474i	0.5742 - 2.1483i	0.5742 - 2.1483i	0.5742 - 2.1484i	0.5742 - 2.1484i
-2.1348 + 0.5737i	-2.1466 + 0.5769i	-2.1475 + 0.5769i	-2.1476 + 0.5769i	-2.1476 + 0.5769i	-2.1476 + 0.5769i
1.5642 + 1.5619i	1.5729 + 1.5705i	1.5734 + 1.5714i	1.5734 + 1.5714i	1.5734 + 1.5714i	1.5734 + 1.5714i
0.5705 - 2.1356i	0.5737 - 2.1474i	0.5742 - 2.1483i	0.5742 - 2.1483i	0.5742 - 2.1484i	0.5742 - 2.1484i
-2.1348 + 0.5737i	-2.1466 + 0.5769i	-2.1475 + 0.5769i	-2.1476 + 0.5769i	-2.1476 + 0.5769i	-2.1476 + 0.5769i
1.5642 + 1.5619i	1.5729 + 1.5705i	1.5734 + 1.5714i	1.5734 + 1.5714i	1.5734 + 1.5714i	1.5734 + 1.5714i

V^2	V^3	V^4	V^5	V^6	V^7
1.0e+003 *	1.0e+003 *	1.0e+003 *	1.0e+003 *	1.0e+003 *	1.0e+003 *
3.8200 + 0.0400i	3.8063 + 0.0398i	3.8053 + 0.0400i	3.8052 + 0.0400i	3.8052 + 0.0400i	3.8052 + 0.0400i
-1.8754 - 3.3282i	-1.8687 - 3.3163i	-1.8680 - 3.3155i	-1.8679 - 3.3154i	-1.8679 - 3.3154i	-1.8679 - 3.3154i
-1.9446 + 3.2882i	-1.9376 + 3.2765i	-1.9373 + 3.2755i	-1.9372 + 3.2754i	-1.9372 + 3.2754i	-1.9372 + 3.2754i
3.7300 + 0.0600i	3.7095 + 0.0597i	3.7079 + 0.0600i	3.7078 + 0.0600i	3.7077 + 0.0600i	3.7077 + 0.0600i
-1.8130 - 3.2603i	-1.8031 - 3.2424i	-1.8020 - 3.2411i	-1.8019 - 3.2410i	-1.8019 - 3.2410i	-1.8019 - 3.2410i
-1.9170 + 3.2003i	-1.9064 + 3.1827i	-1.9059 + 3.1811i	-1.9058 + 3.1810i	-1.9058 + 3.1810i	-1.9058 + 3.1810i
3.7300 + 0.0600i	3.7095 + 0.0597i	3.7079 + 0.0600i	3.7078 + 0.0600i	3.7077 + 0.0600i	3.7077 + 0.0600i
-1.8130 - 3.2603i	-1.8031 - 3.2424i	-1.8020 - 3.2411i	-1.8019 - 3.2410i	-1.8019 - 3.2410i	-1.8019 - 3.2410i
-1.9170 + 3.2003i	-1.9064 + 3.1827i	-1.9059 + 3.1811i	-1.9058 + 3.1810i	-1.9058 + 3.1810i	-1.9058 + 3.1810i
