Institut Teknologi Sepuluh Nopember
Surabaya

JURUSAN TEKNIK FISIKA - FTI

KARAKTERISTIK VARIABEL ACAK –
– OPERATOR E - EKSPEKTASI

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Karakteristik Variabel Acak

Capaian Pembelajaran:

Mampu menentukan karakteristik variable acak dengan menggunakan operator E

Kajian:

1. Definisi Fs Distribusi Komulatif
2. Penentuan besarnya Fs Distribusi Komulatif
3. Sifat Fs Distribusi Komulatif

Karakteristik Variabel Acak
1. Fs Distribusi dan Fs Komulatif

1. $f(x) \geq 0,$
2. $\sum_x f(x) = 1,$
3. $P(X = x) = f(x).$

1. $f(x) \geq 0$, for all $x \in R$.
2. $\int_{-\infty}^{\infty} f(x)\, dx = 1.$
3. $P(a < X < b) = \int_a^b f(x)\, dx.$

$$F(x) = P(X \leq x) = \sum_{t \leq x} f(t), \quad \text{for } -\infty < x < \infty.$$

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\, dt, \quad \text{for } -\infty < x < \infty.$$

2. Karakteristik Var. Acak – operator E

1. Momen ke 1 sd n
2. Momen sentral ke 1 sd n

EKSPEKTASI → E

Sebuah Operator Matematis bersifat linier

Momen ke 1 = Mean untuk Variabel Acak Diskrit

$$\mu = E(X) = \sum_x x f(x) \quad = \quad \mu = E(x) = \sum xp(x).$$

Momen ke 1 = Mean untuk Variabel Acak Kontinyu

$$\mu = E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$$

Momen ke n

Momen ke 1 → E(X) = μ

Momen ke 2 → $E(X^2) = \overline{X^2}$

…

Momen ke n → $E(X^n) = \overline{X^n}$

$$E[X^n] = \overline{X^n} = \begin{cases} \sum_i x_i^n p_X(x_i) & X \text{ discrete} \\ \int_{-\infty}^{\infty} x^n f_X(x)\,dx & X \text{ continuous} \end{cases}$$

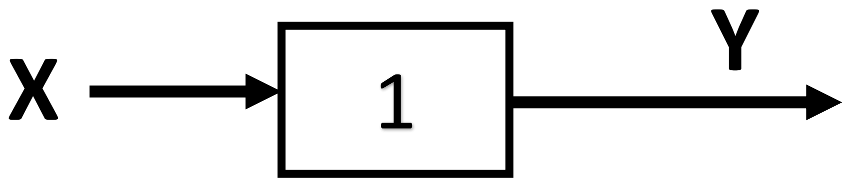
Latihan

Bila tegangan PLN berdistribusi uniform diantara $200 \leq V \leq 240$ Volt. Tegangan PLN sebagai variabel acak kontinyu. Tegangan ini dialirkan pada beban.

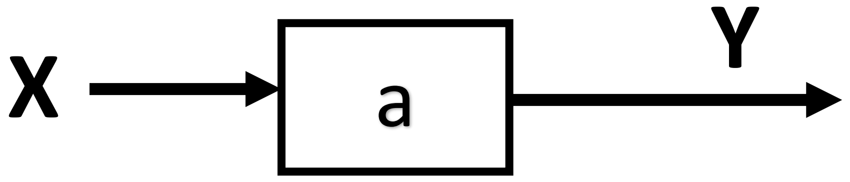
a. Tentukan berapa rata-rata tegangan
b. Tentukan standard deviasi tegangan
c. Tentukan probabilitas beban menerima tegangan ≤ 210 V
d. Tentukan probabilitas beban menerima tegangan $200 \leq V \leq 230$ V
e. Tentukan probabilitas beban menerima tegangan $V \geq 230$ V

Operator E bersifat Linier

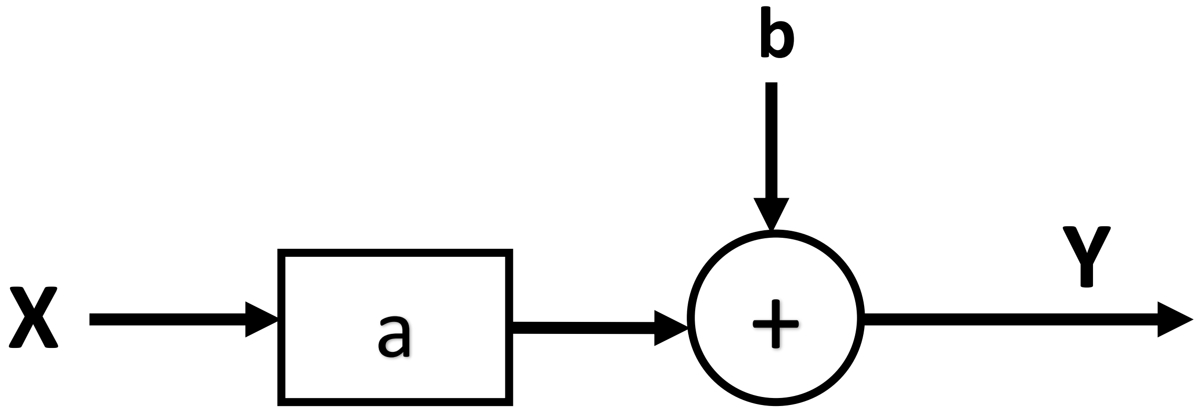
1. $Y = X$



2. $Y = a X$



3. $Y = a X + b$



Momen Sentral

Momen Sentral ke n

$$E[(X-\overline{X})^n] = \overline{(X-\overline{X})^n} = \begin{cases} \sum_i (x_i-\overline{X})^n p_X(x_i) & X \text{ discrete} \\ \int_{-\infty}^{\infty} (x-\overline{X})^n f_X(x)dx & X \text{ continuous} \end{cases}$$

Momen Sentral ke 2

Variansi

$$\sigma_X^2 = E[(X-\overline{X})^2] = \overline{(X-\overline{X})^2} = \begin{cases} \sum_i (x_i-\overline{X})^2 p_X(x_i) & X \text{ discrete} \\ \int_{-\infty}^{\infty} (x-\overline{X})^2 f_X(x)dx & X \text{ continuous} \end{cases}$$

$$\text{Mean}: \ E(X) = \mu = \sum_{\text{all } x} yp(x)$$

$$\text{Mean dari fungsi } g(X): \quad E[g(X)] = \sum_{\text{all } x} g(x)p(x) = \sum_{\text{all } x} g(x)f(x)$$

$$\text{Variansi}: \ V(X) = \sigma^2 = E\left[(X - E(X))^2\right] = E\left[(X-\mu)^2\right]$$

$$= \sum_{\text{all } x} (x-\mu)^2 p(x) = \sum_{\text{all } y} \left(x^2 - 2x\mu + \mu^2\right) p(x)$$

$$= \sum_{\text{all x}} x^2 p(x) - 2\mu \sum_{\text{all } x} xp(x) + \mu^2 \sum_{\text{all } x} p(x)$$

$$= E\left[X^2\right] - 2\mu(\mu) + \mu^2(1) = E\left[X^2\right] - \mu^2$$

$$\text{Standard Deviasi}: \sigma = +\sqrt{\sigma^2}$$

Contoh variabel acak berdistribusi Normal

Momen ke 1 = Mean

Untuk Var. acak berdistribusi Uniform diar a – b, dimana letak Mean nya?

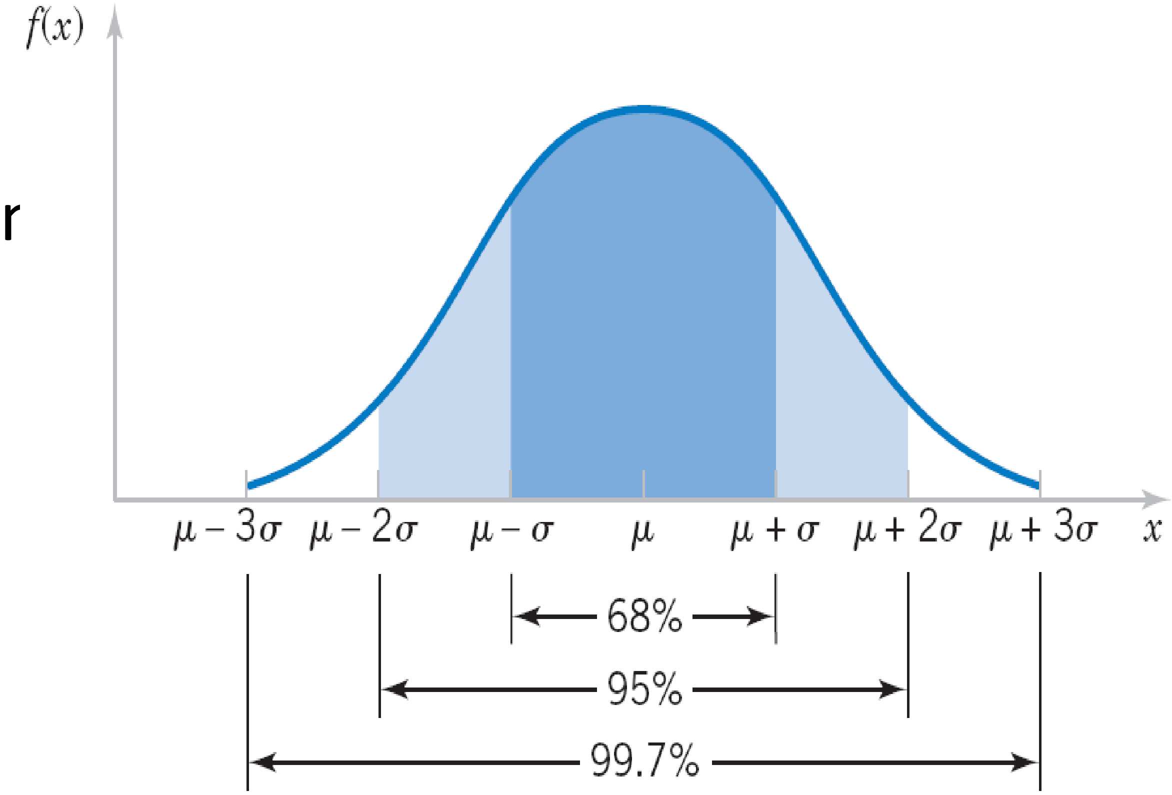


Figure 3-13 Probabilities associated with a normal distribution.

Misalkan, terdapat sebuah Variabel Acak X diskrit dengan fungsi distribusi probabilitas berikut ini:

X	0	1	2	3	4	5
f(x)	1/6	1/6	1/6	1/6	1/6	1/6

Maka: $$\mu = E(X) = \sum_x x f(x)$$

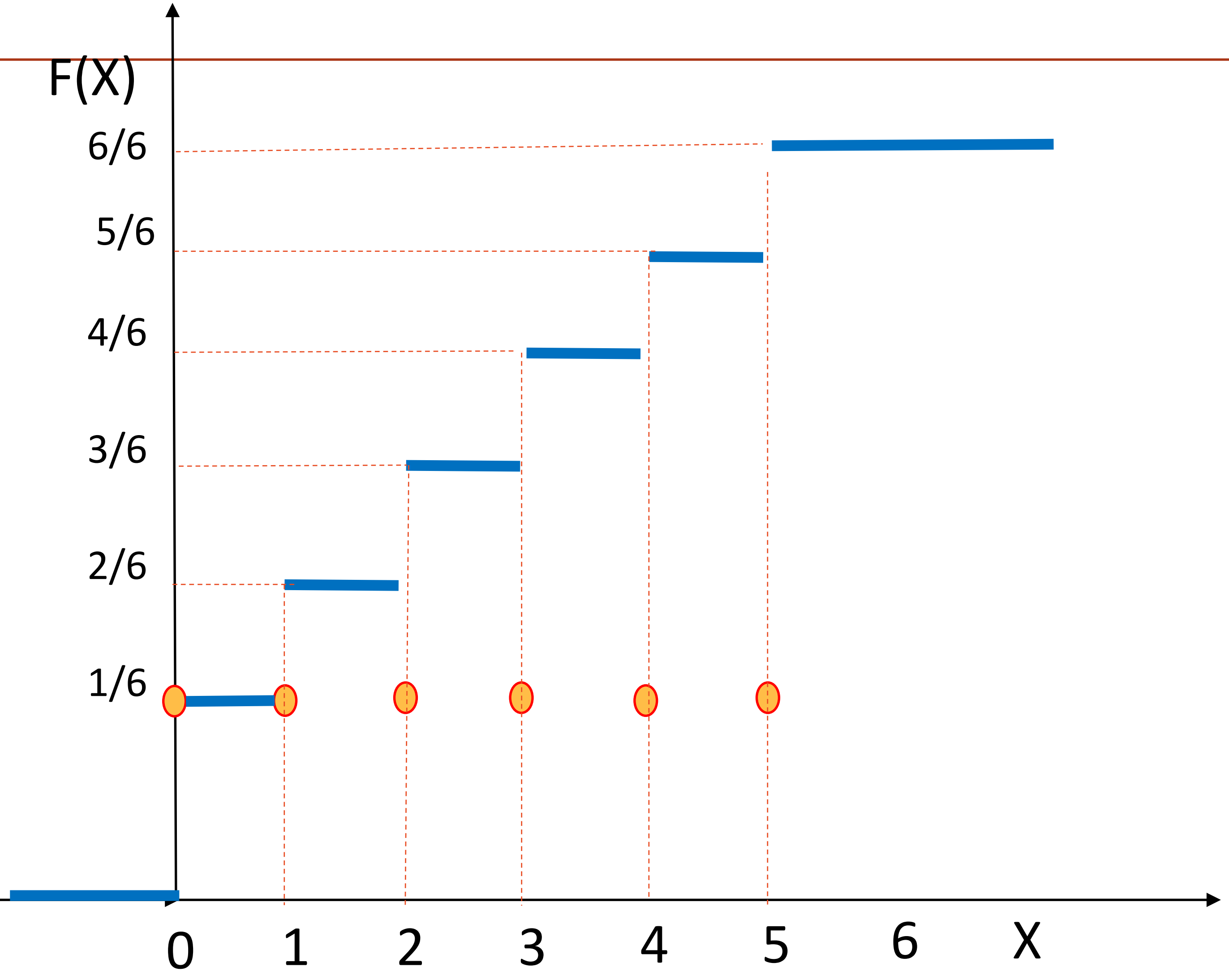
μ = 0.(1/6) + 1.(1/6) + 2. (1/6) + 3. (1/6) + 4.(1/6) + 5.(1/6)

μ = (1/6) + (2/6) + (3/6) + (4/6) + (5/6)

μ = 15/6 = 2.5

Berdistribusi UNIFORM

Distribusi Var. Diskrit / Kontinyu ?

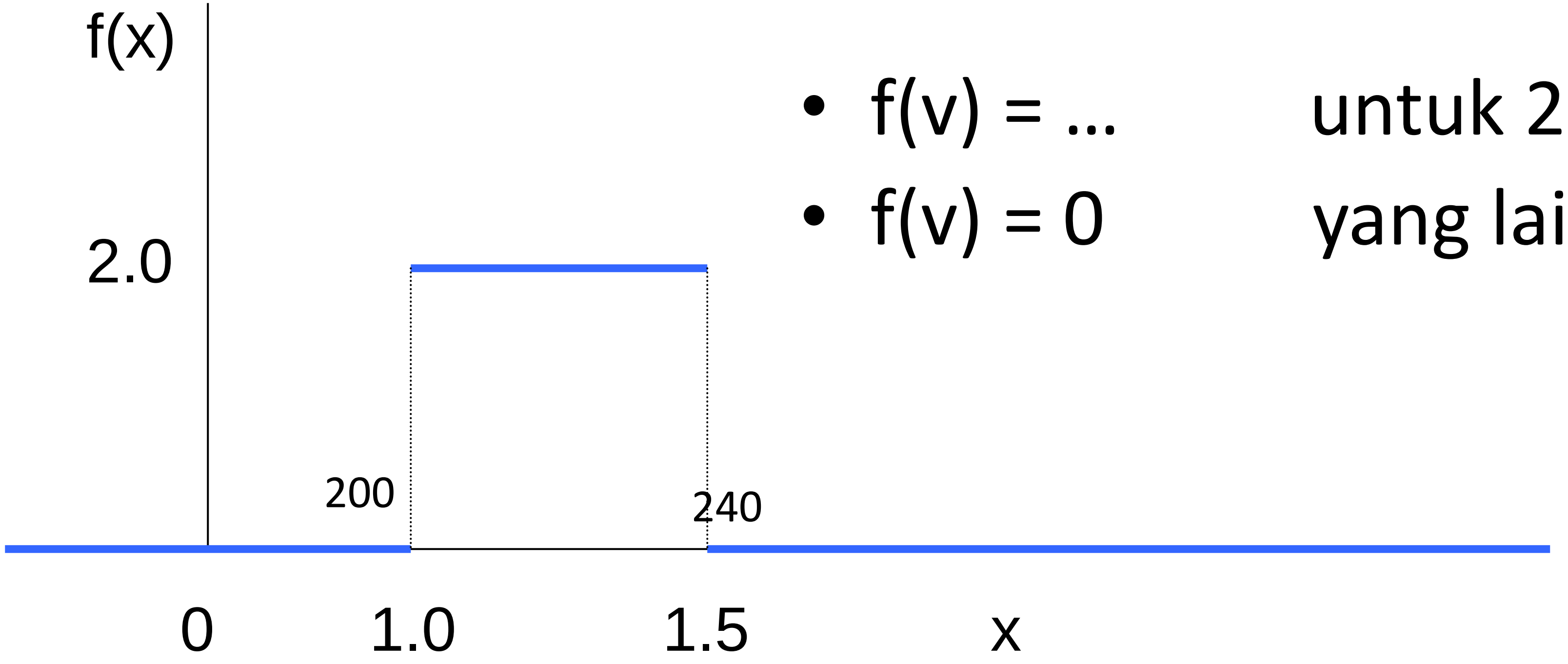


Materi – Contoh Distribusi Uniform

- $f(x) = 2$ untuk $1 \leq x \leq 1.5$ dan
- $f(x) = 0$ yang lain

Fungsi tidak pernah negatif

Luasan di bawah kurva = (1/2) (2) = 1.

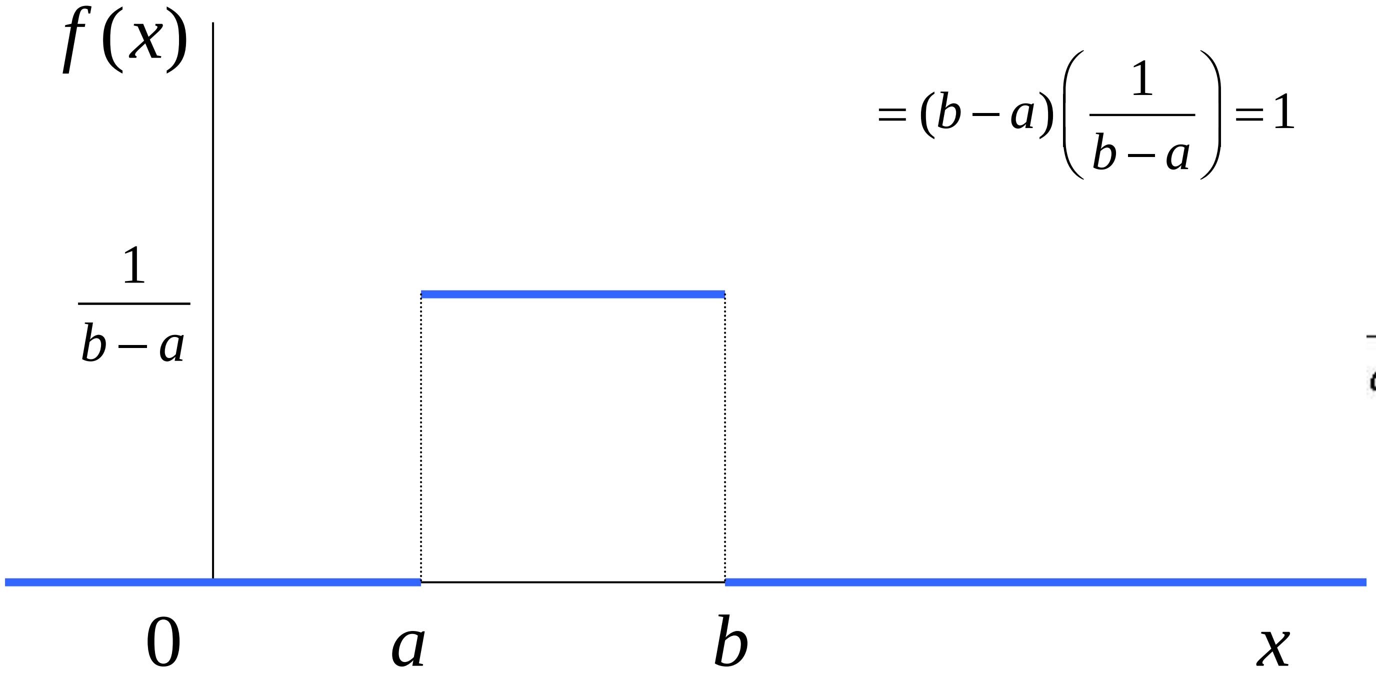


- $f(v) = \ldots$ untuk $200 \leq x \leq 240$ dan
- $f(v) = 0$ yang lain

Materi – Distribusi Uniform

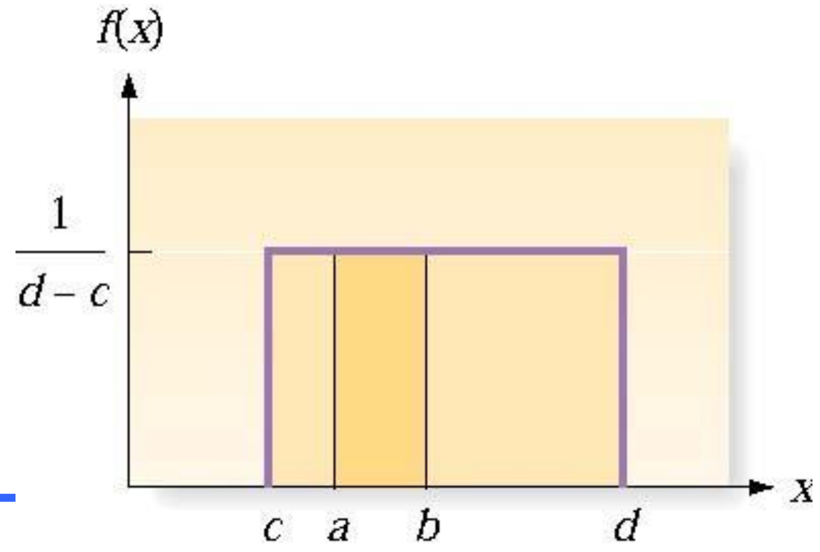
$$f(x) = \frac{1}{b-a} \qquad \text{if } a \le x \le b$$

$$f(x) = 0 \qquad \text{otherwise}$$



Luasan di bawah kurva = 1

$$= (b-a)\left(\frac{1}{b-a}\right) = 1$$



Materi – Distribusi Uniform

Mean dan Variansi

mean: $$\mu = \frac{a+b}{2}$$

variance: $$\sigma^2 = \frac{(b-a)^2}{12}$$

standard deviation: $$\sigma = \sqrt{\frac{(b-a)^2}{12}}$$

Materi

Example 4.3: Let X be the random variable that denotes the life in hours of a certain electronic device. The probability density function is

$$f(x) = \begin{cases} \frac{20,000}{x^3}, & x > 100, \\ 0, & \text{elsewhere}. \end{cases}$$

Find the expected life of this type of device.

***Solution*:** Using Definition 4.1, we have

$$\mu = E(X) = \int_{100}^{\infty} x \frac{20,000}{x^3}\, dx = \int_{100}^{\infty} \frac{20,000}{x^2}\, dx = 200.$$

Therefore, we can expect this type of device to last, *on average*, 200 hours.

Sifat Operator E

1. Linier

$$E(aX) = aE(X)$$
$$= a\mu$$
$$= a\bar{X}$$

$$E(aX + bY) = E(aX) + E(bY)$$
$$= aE(x) + bE(Y)$$
$$= a\bar{X} + b\bar{Y}$$

Sifat Operator E

Latihan

Diketahui sebuah fungsi distribusi probabilitas

$$f(x) = \begin{cases} x, & 0 < x < 1, \\ 2 - x, & 1 \le x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

Tentukan

1. Momen ke 1
2. Momen ke 2
3. Momen ke 3

Sifat Operator E

Latihan

Diketahui sebuah fungsi distribusi probabilitas

3.13 The probability distribution of X, the number of imperfections per 10 meters of a synthetic fabric in continuous rolls of uniform width, is given by

x	0	1	2	3	4
$f(x)$	0.41	0.37	0.16	0.05	0.01

Tentukan

1. Momen ke 1
2. Momen ke 2
3. Momen ke 3