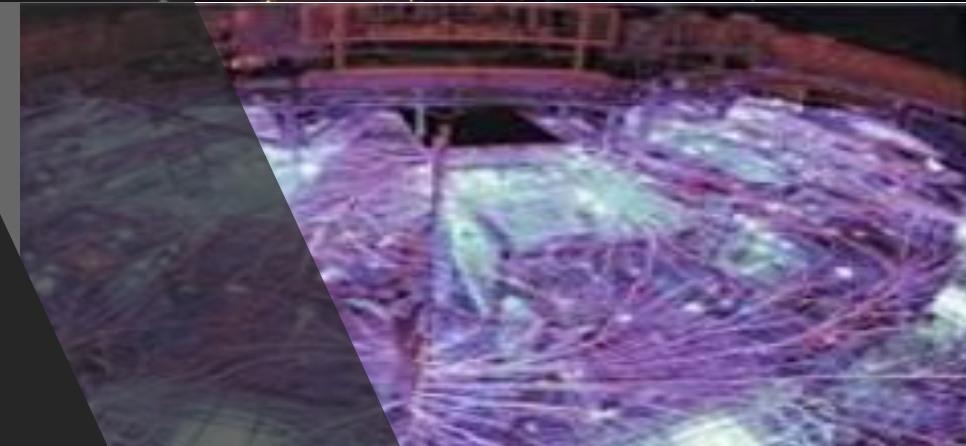


# PERSAMAAN DIFFERENSIAL

**PD Exact**

MATEMATIKA  
REKAYASA 1

AULIA SITI AISJAH – TEKNIK FISIKA ITS



# PD Exact

Bentuk PD

$$M(x, y) + N(x, y)y' = 0$$

Bila ada fs  $\psi$  sehingga

$$\psi_x(x, y) = M(x, y), \quad \psi_y(x, y) = N(x, y)$$



$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Dan  $\psi(x, y) = c$  dan

dideff.  $y = \phi(x)$  maka

$$M(x, y) + N(x, y)y' = \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} \frac{dy}{dx} = \frac{d}{dx} \psi[x, \phi(x)]$$

$$\frac{d}{dx} \psi[x, \phi(x)] = 0$$

Bentuk PD menjadi

*Sehingga*  $\psi(x, y) = c$  merupakan solusi PD di atas

**Dalam kasus ini PD dikatakan PD exact.**

# Teori 1

Jika PD dituliskan

$$M(x, y) + N(x, y)y' = 0 \quad (1)$$

Dimana fungsi  $M$ ,  $N$ ,  $M_y$  and  $N_x$  semua kontinyu

Dan daerah yang memenuhi  $R: (x, y) \in (\alpha, \beta) \times (\gamma, \delta)$ .

Pers (1) dikatakan sbg PD exact, jika dan hanya jika

$$M_y(x, y) = N_x(x, y), \quad \forall (x, y) \in R \quad (2)$$

Jika ada fungsi  $\psi$  yg memenuhi kondisi

$$\psi_x(x, y) = M(x, y), \quad \psi_y(x, y) = N(x, y) \quad (3)$$

Maka  $M$  dan  $N$  adalah solusi (2).

# Contoh 1

✦ PD berikut

$$\frac{dy}{dx} = -\frac{x+4y}{4x-y} \Leftrightarrow (x+4y) + (4x-y)y' = 0$$

✦ Disini

$$M(x, y) = x + 4y, N(x, y) = 4x - y$$

dan

$$M_y(x, y) = 4 = N_x(x, y) \Rightarrow PD \text{ exact}$$

✦ Berdasar teori

$$\psi_x(x, y) = x + 4y, \psi_y(x, y) = 4x - y$$

✦ Maka

$$\psi(x, y) = \int \psi_x(x, y) dx = \int (x + 4y) dx = \frac{1}{2} x^2 + 4xy + C(y)$$

# Solusi

✦ Bila

$$\psi_x(x, y) = x + 4y, \quad \psi_y(x, y) = 4x - y$$

dan

$$\psi(x, y) = \int \psi_x(x, y) dx = \int (x + 4y) dx = \frac{1}{2} x^2 + 4xy + C(y)$$

✦ Diikuti oleh

$$\psi_y(x, y) = 4x - y = 4x + C'(y) \Rightarrow C'(y) = -y \Rightarrow C(y) = -\frac{1}{2} y^2 + k$$

✦ Maka

$$\psi(x, y) = \frac{1}{2} x^2 + 4xy - \frac{1}{2} y^2 + k$$

✦ Berdasar teorema

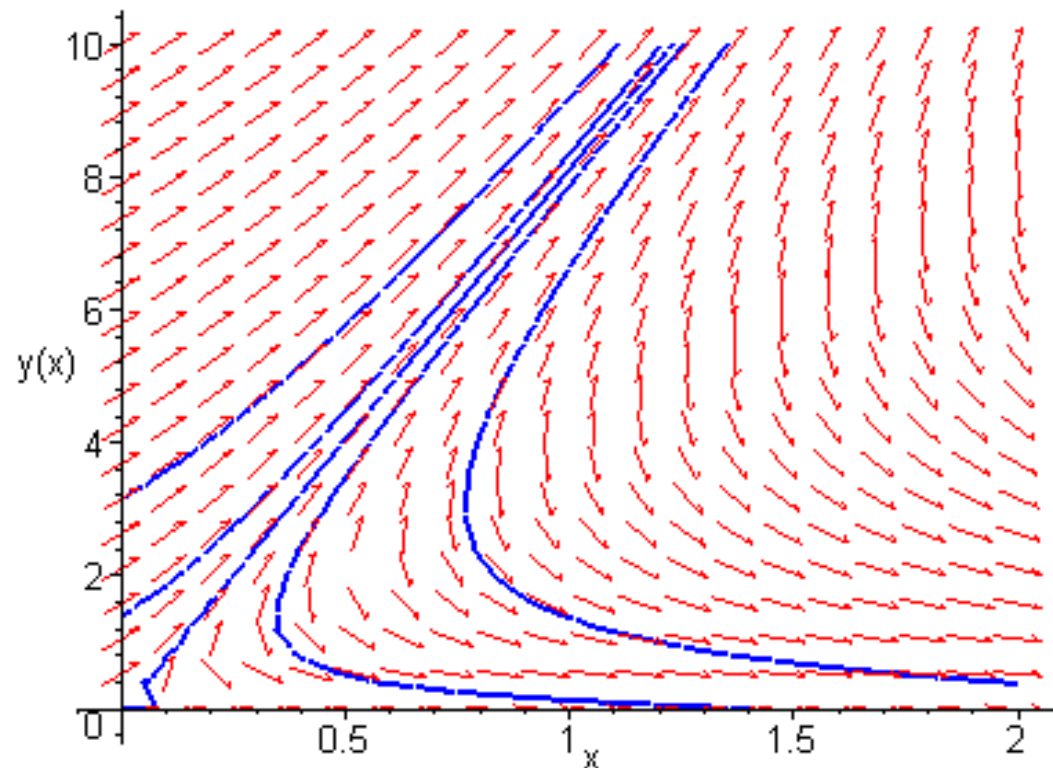
$$x^2 + 8xy - y^2 = c$$

# Contoh 1 - Peny. Bentuk kurva

✿ Bentuk PD dan solusi nya

$$\frac{dy}{dx} = -\frac{x+4y}{4x-y} \Leftrightarrow (x+4y) + (4x-y)y' = 0 \Rightarrow x^2 + 8xy - y^2 = c$$

✿ Garifk yg menunjukkan arah dr medan dr PD tersebut

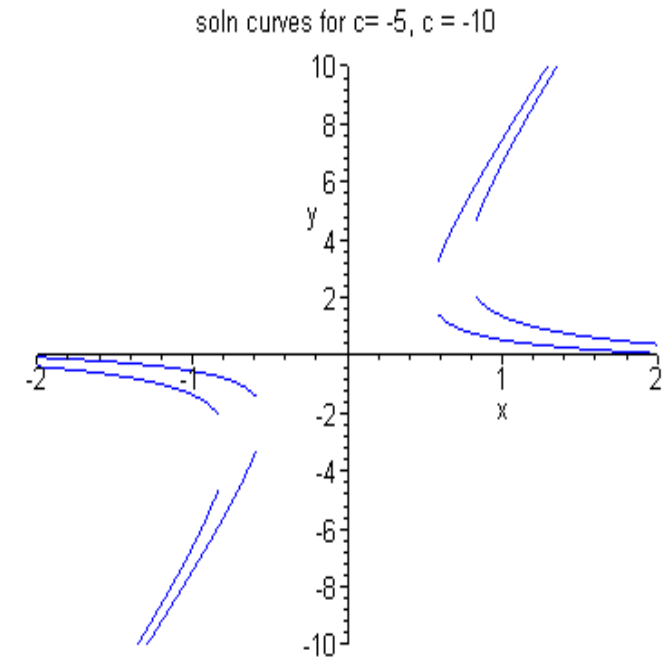
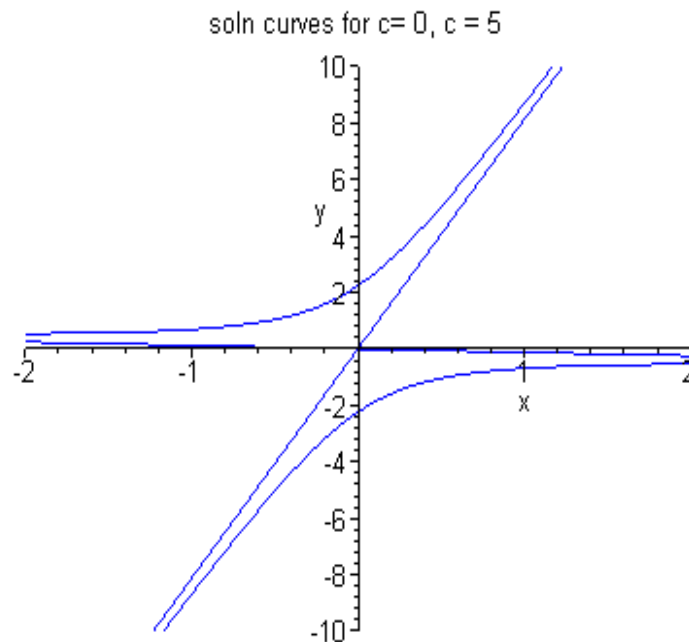
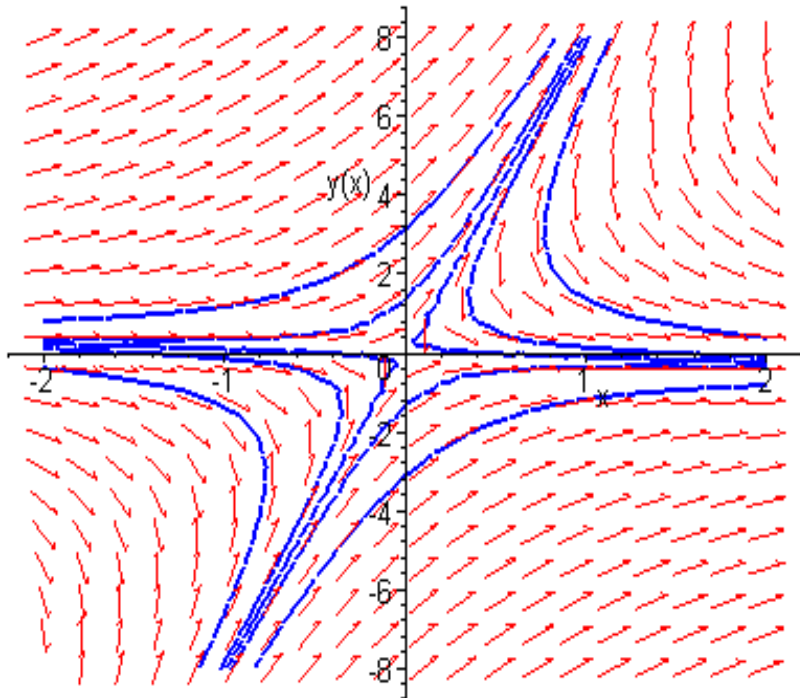


# Contoh 1: Peny. Eksplisit dan grafik

✦ Bentuk solusi PD  $x^2 + 8xy - y^2 = c$

✦ Pada kasus ini,  $y^2 - 8xy - x^2 - c = 0 \Rightarrow y = 4x \pm \sqrt{17x^2 + c}$

✦ Beberapa kurva dengan nilai c yg berbeda



## Contoh 2: PD Exact

✿ PD berikut  $(y \cos x + 2xe^y) + (\sin x + x^2e^y - 1)y' = 0$

✿ Maka  $M(x, y) = y \cos x + 2xe^y$ ,  $N(x, y) = \sin x + x^2e^y - 1$

dan

$$M_y(x, y) = \cos x + 2xe^y = N_x(x, y) \Rightarrow PD \text{ exact}$$

✿ Berdasar teori,  $\psi_x(x, y) = M = y \cos x + 2xe^y$ ,  $\psi_y(x, y) = N = \sin x + x^2e^y - 1$

✿ Dan 
$$\psi(x, y) = \int \psi_x(x, y) dx = \int (y \cos x + 2xe^y) dx = y \sin x + x^2e^y + C(y)$$



## Contoh 2

✦ Terdapat

$$\psi_x(x, y) = M = y \cos x + 2xe^y, \quad \psi_y(x, y) = N = \sin x + x^2e^y - 1$$

dan

$$\psi(x, y) = \int \psi_x(x, y) dx = \int (y \cos x + 2xe^y) dx = y \sin x + x^2e^y + C(y)$$

✦ diikuti

$$\psi_y(x, y) = \sin x + x^2e^y - 1 = \sin x + x^2e^y + C'(y)$$

$$\Rightarrow C'(y) = -1 \Rightarrow C(y) = -y + k$$

✦ Maka

$$\psi(x, y) = y \sin x + x^2e^y - y + k$$

✦ Berdasar teori

$$y \sin x + x^2e^y - y = c$$

## Contoh 2 dan kurva

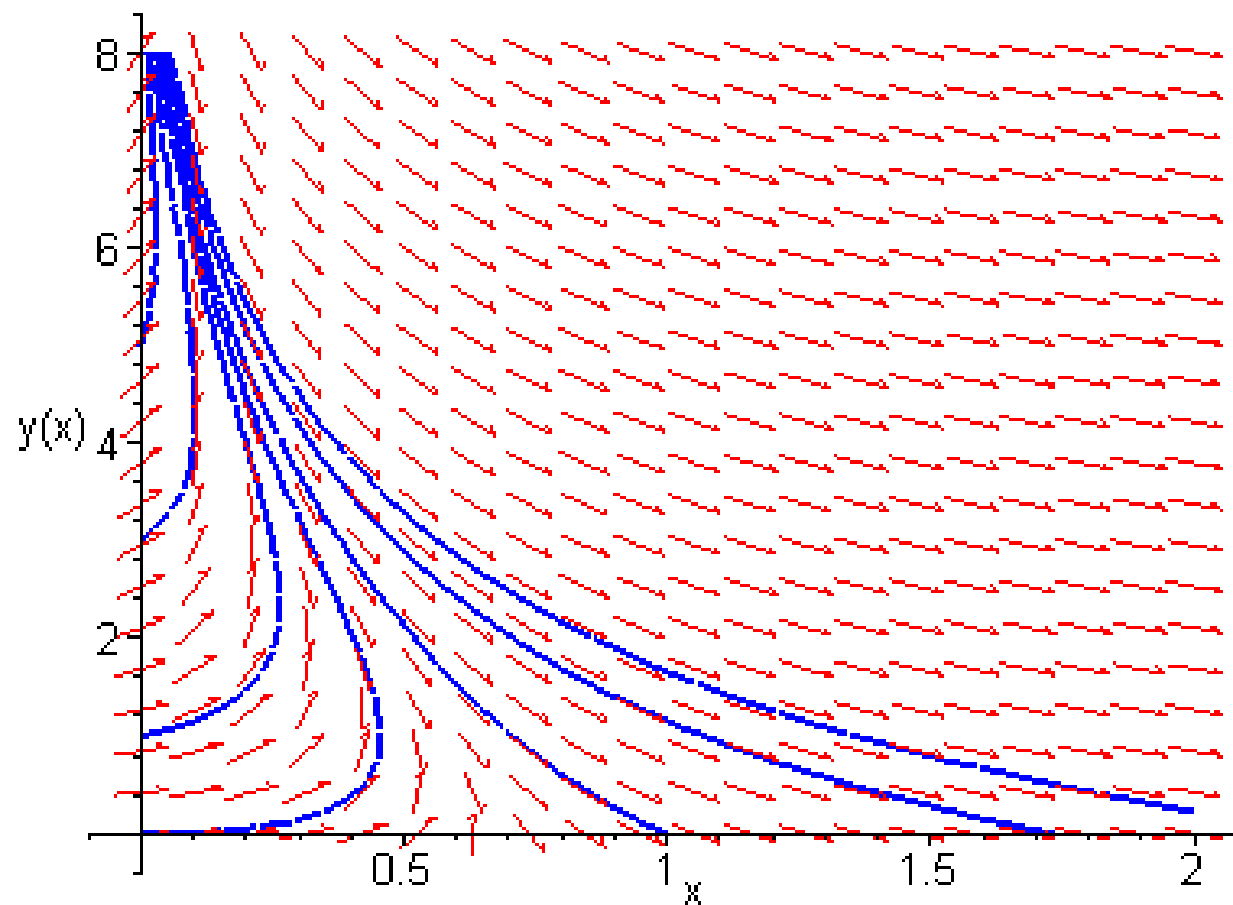
✦ Bentuk PD

$$(y \cos x + 2xe^y) + (\sin x + x^2e^y - 1)y' = 0,$$

✦ solusi

$$y \sin x + x^2e^y - y = c$$

✦ Grafik solusi.



## Contoh 3: Pers. Non exact → Gunakan Faktor Pengintegrasi

✦ Bentuk PD

$$(3xy + y^2) + (2xy + x^3)y' = 0$$

Bila

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

✦ Maka

$$M(x, y) = 3xy + y^2, N(x, y) = 2xy + x^3$$

dan

$$M_y(x, y) = 3x + 2y \neq 2y + 3x^2 = N_x(x, y) \Rightarrow PD \text{ bukan exact}$$

✦ Utk menunjukkan bahwa PD bukan exact, disini  $\psi$

✦ maka

$$\psi_x(x, y) = M = 3xy + y^2, \psi_y(x, y) = N = 2xy + x^3$$

$$\psi(x, y) = \int \psi_x(x, y) dx = \int (3xy + y^2) dx = 3x^2 y / 2 + xy^2 + C(y)$$

## Contoh 3: PD bukan exact

✿ Fungsi  $\psi$

$$\psi_x(x, y) = M = 3xy + y^2, \quad \psi_y(x, y) = N = 2xy + x^3$$

dan

$$\psi(x, y) = \int \psi_x(x, y) dx = \int (3xy + y^2) dx = 3x^2 y / 2 + xy^2 + C(y)$$

✿ Maka

$$\begin{aligned} \psi_y(x, y) &= 2xy + x^3 = 3x^2/2 + 2xy + C'(y) \\ \Rightarrow C'(y) &= x^3 - \frac{3x^2}{2} \Rightarrow C(y) = x^3 y - 3x^2 y / 2 + k \end{aligned}$$

✿ Disini nilai  $\psi$ .

$$x^3 y + xy^2 = c$$

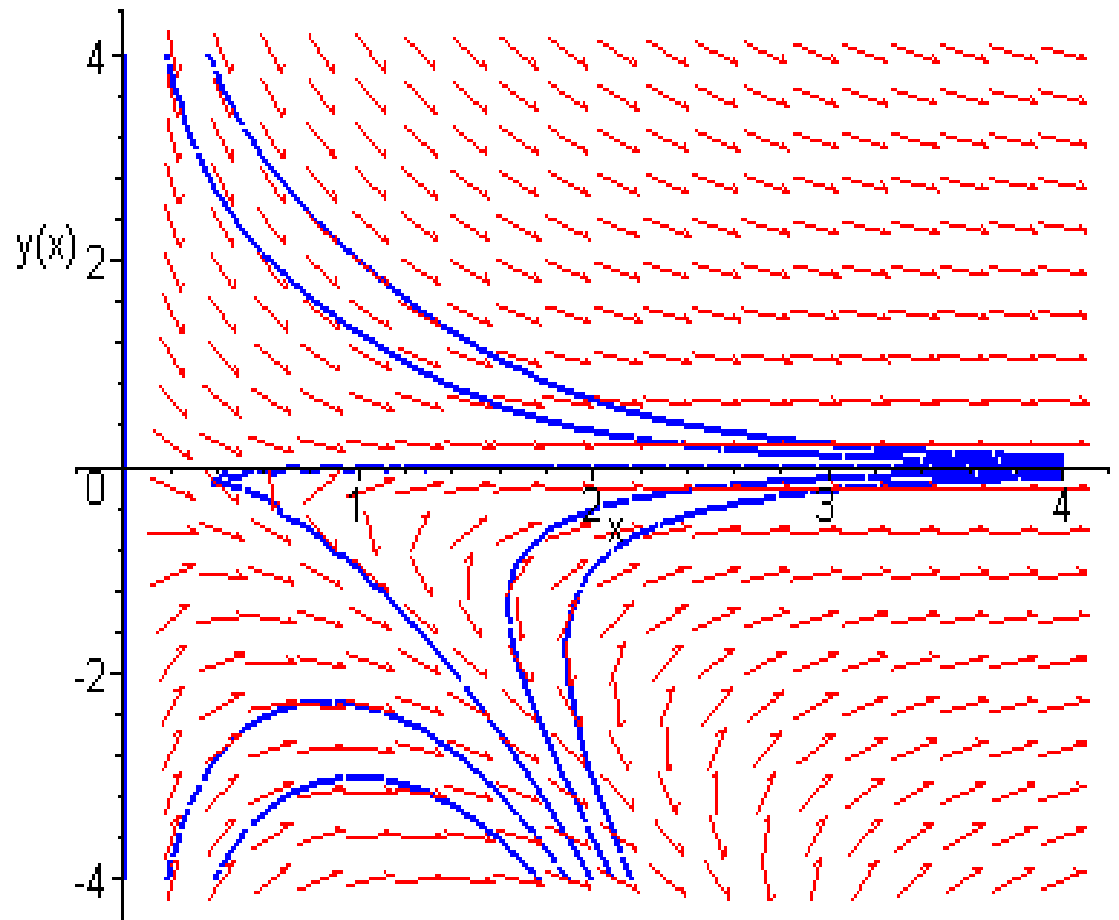
sesuai dengan yang didefinisikan pada  $y$ , merupakan bukan solusi PD

## Contoh 3: Grafik

☀ PD dan solusi  $(3xy + y^2) + (2xy + x^3)y' = 0,$   
 $x^3y + xy^2 = c$

☀ Plot grafik dari y

☀ Dari grafik, terlihat y tidak memenuhi PD



# Factor PengIntegrasi (FI)

- Beberapa PD non exact dapat dijadikan PD exact dengan mengalikan dg faktor  $\mu(x, y)$ :

$$M(x, y) + N(x, y)y' = 0$$

$$\mu(x, y)M(x, y) + \mu(x, y)N(x, y)y' = 0$$

- Untuk menjadi PD exact, perlu

$$(\mu M)_y = (\mu N)_x \Leftrightarrow M\mu_y - N\mu_x + (M_y - N_x)\mu = 0$$

- Peny scr parsial akan sulit, maka jika  $\mu$  adalah fs dr  $x$  dan  $\mu_y = 0$  dapat diperoleh

$$\frac{d\mu}{dx} = \frac{M_y - N_x}{N} \mu,$$

Terlihat sisi kana hanya  $x$  dan juga  $\mu$  adalah fs dr  $y$  saja.

## Contoh 4: PD non exact

☀ Perhatikan PD non exact

$$(3xy + y^2) + (x^2 + xy)y' = 0$$

☀ Dengan FI

$$\frac{d\mu}{dx} = \frac{M_y - N_x}{N} \mu \Leftrightarrow \frac{d\mu}{dx} = \frac{\mu}{x} \Rightarrow \mu(x) = x$$

☀ Kalikan PD dengan  $\mu$ ,

☀ diperoleh PD exact

dan solusinya

$$(3x^2y + xy^2) + (x^3 + x^2y)y' = 0,$$

$$x^3y + \frac{1}{2}x^2y^2 = c$$

## Tugas 2:

**NRP Ganjil – 3 soal Ganjil (soal 1-10)+ 2 soal Ganjil (17-26)**

**NRP Genap – 3 soal Genap (soal 1-10)+ 2 soal Genap (17-26)**

1.  $(2x - 3y) dx + (2y - 3x) dy = 0$

2.  $ye^x dx + e^x dy = 0$

3.  $(3y^2 + 10xy^2) dx + (6xy - 2 + 10x^2y) dy = 0$

4.  $2 \cos(2x - y) dx - \cos(2x - y) dy = 0$

5.  $(4x^3 - 6xy^2) dx + (4y^3 - 6xy) dy = 0$

6.  $2y^2e^{xy^2} dx + 2xye^{xy^2} dy = 0$

7.  $\frac{1}{x^2 + y^2} (x dy - y dx) = 0$

8.  $e^{-(x^2+y^2)}(x dx + y dy) = 0$

9.  $\frac{1}{(x - y)^2} (y^2 dx + x^2 dy) = 0$

10.  $e^y \cos xy [y dx + (x + \tan xy) dy] = 0$

17.  $y dx - (x + 6y^2) dy = 0$

18.  $(2x^3 + y) dx - x dy = 0$

19.  $(5x^2 - y) dx + x dy = 0$

20.  $(5x^2 - y^2) dx + 2y dy = 0$

21.  $(x + y) dx + \tan x dy = 0$

22.  $(2x^2y - 1) dx + x^3 dy = 0$

23.  $y^2 dx + (xy - 1) dy = 0$

24.  $(x^2 + 2x + y) dx + 2 dy = 0$

25.  $2y dx + (x - \sin \sqrt{y}) dy = 0$

26.  $(-2y^3 + 1) dx + (3xy^2 + x^3) dy = 0$



**Tugas 2 dikumpulkan paling lambat  
12 Oktober 2020, mell MyClassroom**

