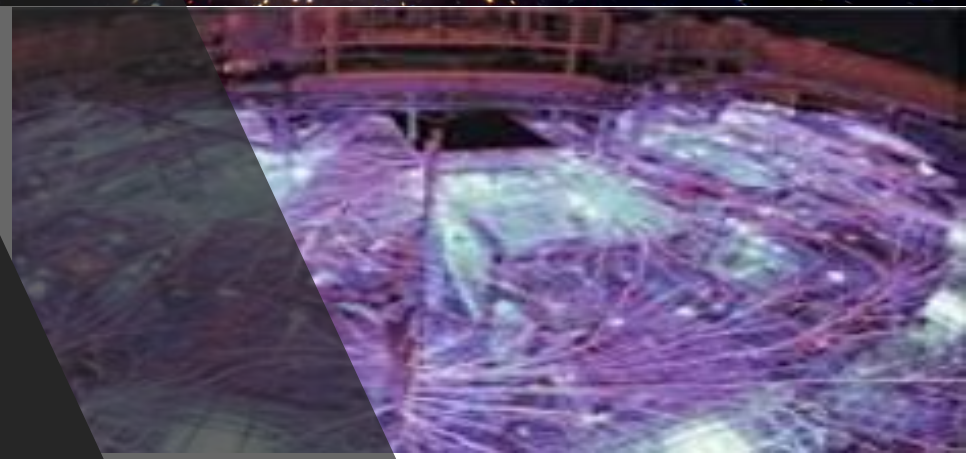
PERSAMAAN ALJABAR LINIER

PENYELESAIAN DG MET. ELIMINASI GAUSS

MATEMATIKA
REKAYASA 1

AULIA SITI AISJAH – TEKNIK FISIKA ITS



Pers. Aljabar Linier

Capaian Pembelajaran:

- Mampu menggunakan hukum fisika yang berlaku pada sistem dinamik - dan menyusun nya dalam bentuk persamaan aljabar linier.
- Mampu menyelesaikan persamaan aljabar linier dengan metode eliminasi Gauss

Kajian:

1. **Metode Eliminasi Gauss Forward**
2. **Metode Eliminasi Gauss Backward**

Eliminasi Gauss

Bentuk persamaan matriks

A X= C

Dua tahap penyelesaian

1. Forward Elimination
2. Back Substitution

Eliminasi Maju - Forward

Tujuan → membentuk matriks dalam bentuk segitiga atas

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ -96.21 \\ 0.735 \end{bmatrix}$$

Eliminasi Forwad

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n = b_2$$

$$\vdots \qquad \vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \ldots + a_{nn}x_n = b_n$$

Ada (*n-1*) step

Eliminasi forward

Step 1

Untuk pers. brs 2, bagi dengan pers. brs 1 a_{11} dan kalikan dg a_{21}

$$\left[\frac{a_{21}}{a_{11}}\right](a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1)$$

$$a_{21}x_1 + \frac{a_{21}}{a_{11}}a_{12}x_2 + \ldots + \frac{a_{21}}{a_{11}}a_{1n}x_n = \frac{a_{21}}{a_{11}}b_1$$

Eliminasi Gauss

Kurangkan dg pers. Brs 2

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n = b_2$$

$$- \quad a_{21}x_1 + \frac{a_{21}}{a_{11}}a_{12}x_2 + \ldots + \frac{a_{21}}{a_{11}}a_{1n}x_n = \frac{a_{21}}{a_{11}}b_1$$

$$\left(a_{22} - \frac{a_{21}}{a_{11}}a_{12}\right)x_2 + \ldots + \left(a_{2n} - \frac{a_{21}}{a_{11}}a_{1n}\right)x_n = b_2 - \frac{a_{21}}{a_{11}}b_1$$

or $\quad a'_{22}x_2 + \ldots + a'_{2n}x_n = b'_2$

Eliminasi Gauss

Ulangi prosedur (pd step 1) utk brs – brs berikutnya

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1$$
$$a'_{22}x_2 + a'_{23}x_3 + \ldots + a'_{2n}x_n = b'_2$$
$$a'_{32}x_2 + a'_{33}x_3 + \ldots + a'_{3n}x_n = b'_3$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$a'_{n2}x_2 + a'_{n3}x_3 + \ldots + a'_{nn}x_n = b'_n$$

End - step 1

Eliminasi Forward

Step 2

Ulangi cara di step 1 untuk brs ke 3, 4 dst.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1$$

$$a'_{22}x_2 + a'_{23}x_3 + \ldots + a'_{2n}x_n = b'_2$$

$$a''_{33}x_3 + \ldots + a''_{3n}x_n = b''_3$$

$$\vdots \qquad \vdots$$

$$a''_{n3}x_3 + \ldots + a''_{nn}x_n = b''_n$$

End of Step 2

Eliminasi Forward

Sapai pada step ke (n-1)

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n &= b_1 \\
a_{22}^{'}x_2 + a_{23}^{'}x_3 + \ldots + a_{2n}^{'}x_n &= b_2^{'} \\
a_{33}^{''}x_3 + \ldots + a_{3n}^{''}x_n &= b_3^{''} \\
&\vdots \\
a_{nn}^{(n-1)}x_n &= b_n^{(n-1)}
\end{aligned}
$$

End Step (n-1)

Akhir dari step dalam eliminasi – menghasilkan matrik

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & a_{22}^{'} & a_{23}^{'} & \cdots & a_{2n}^{'} \\ 0 & 0 & a_{33}^{''} & \cdots & a_{3n}^{''} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & 0 & a_{nn}^{(n-1)} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2^{'} \\ b_3^{''} \\ \vdots \\ b_n^{(n-1)} \end{bmatrix}$$

Substitusi balik

Untuk menyelesaikan bentuk pers. Matrik di bawah – lakukan secara balik – perlakuan eliminasi

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ -96.21 \\ 0.735 \end{bmatrix}$$

contoh

Substitusi balik

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1$$

$$a_{22}^{'}x_2 + a_{23}^{'}x_3 + \ldots + a_{2n}^{'}x_n = b_2^{'}$$

$$a_{33}^{''}x_3 + \ldots + a_{n}^{''}x_n = b_3^{''}$$

$$\vdots \qquad \vdots$$

$$a_{nn}^{(n-1)}x_n = b_n^{(n-1)}$$

Substitusi balik

Mulai dengan persamaan baris terakhir

$$x_n = \frac{b_n^{(n-1)}}{a_{nn}^{(n-1)}}$$

Substitusi balik

$$x_n = \frac{b_n^{(n-1)}}{a_{nn}^{(n-1)}}$$

$$x_i = \frac{b_i^{(i-1)} - a_{i,i+1}^{(i-1)} x_{i+1} - a_{i,i+2}^{(i-1)} x_{i+2} - \ldots - a_{i,n}^{(i-1)} x_n}{a_{ii}^{(i-1)}} \text{ for } i = n-1,\ldots,1$$

$$x_i = \frac{b_i^{(i-1)} - \sum_{j=i+1}^{n} a_{ij}^{(i-1)} x_j}{a_{ii}^{(i-1)}} \text{ for } i = n-1,\ldots,1$$

contoh 1

Kecepatan ke atas – sebuah roket

Table 1 Velocity vs. waktu

Time, $t\,(\mathrm{s})$	**Velocity,** $v\,(\mathrm{m/s})$
5	106.8
8	177.2
12	279.2

Pers. Kecepatan didekati dengan bentuk :

$$v(t) = a_1 t^2 + a_2 t + a_3 , \qquad 5 \le \mathrm{t} \le 12.$$

Tentukan saat t=6 second → kecepatan roket tsb.

Assume

$$v(t) = a_1 t^2 + a_2 t + a_3 \ , \ \ 5 \le t \le 12.$$

Hasil bentuk persamaan matrik, dengan memasukkan data dari Tabel:

$$\begin{bmatrix} t_1^2 & t_1 & 1 \\ t_2^2 & t_2 & 1 \\ t_3^2 & t_3 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Saat t = 5:

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

Lakukan eliminasi dengan forward dan kemudian backward

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix} \Rightarrow \left[\begin{array}{ccc:c} 25 & 5 & 1 & 106.8 \\ 64 & 8 & 1 & 177.2 \\ 144 & 12 & 1 & 279.2 \end{array}\right]$$

Bentuk matrik - augmented

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix} \Rightarrow \left[\begin{array}{ccc:c} 25 & 5 & 1 & 106.8 \\ 64 & 8 & 1 & 177.2 \\ 144 & 12 & 1 & 279.2 \end{array}\right]$$

Bentuk matrik - augmented

Forward Elimination

Jumlah step dalam forward – dapat diperoleh dari orde matrik (orde matrik dalam contoh n =3)

Jumlah step eliminasi forward

(n-1)=(3-1)=2

Step 1

$$\left[\begin{array}{ccc:c} 25 & 5 & 1 & 106.8 \\ 64 & 8 & 1 & 177.2 \\ 144 & 12 & 1 & 279.2 \end{array}\right]$$

Bagi pers.1 dg 25 dan

Kalikan dg 64, $\frac{64}{25} = 2.56$

$$\left[\begin{array}{ccc:c} 25 & 5 & 1 & 106.8 \end{array}\right] \times 2.56 = \left[\begin{array}{ccc:c} 64 & 12.8 & 2.56 & 273.408 \end{array}\right]$$

Kurangkan hasilnya dg pers. 2

$$\begin{array}{r} \left[\begin{array}{ccc:c} 64 & 8 & 1 & 177.2 \end{array}\right] \\ -\left[\begin{array}{ccc:c} 64 & 12.8 & 2.56 & 273.408 \end{array}\right] \\ \hline \left[\begin{array}{ccc:c} 0 & -4.8 & -1.56 & -96.208 \end{array}\right] \end{array}$$

Substitusi pers. 2

$$\left[\begin{array}{ccc:c} 25 & 5 & 1 & 106.8 \\ 0 & -4.8 & -1.56 & -96.208 \\ 144 & 12 & 1 & 279.2 \end{array}\right]$$

$$\left[\begin{array}{ccc:c} 25 & 5 & 1 & 106.8 \\ 0 & -4.8 & -1.56 & -96.208 \\ 144 & 12 & 1 & 279.2 \end{array}\right]$$

Bagi pers.1 dg 25 dan

Kalikan dg 144, $\frac{144}{25} = 5.76$

$$\left[\begin{array}{ccc:c} 25 & 5 & 1 & 106.8 \end{array}\right] \times 5.76 = \left[\begin{array}{ccc:c} 144 & 28.8 & 5.76 & 615.168 \end{array}\right]$$

Kurangkan hasil nya dg pers.3

$$\begin{array}{r} \left[\begin{array}{ccc:c} 144 & 12 & 1 & 279.2 \end{array}\right] \\ -\left[\begin{array}{ccc:c} 144 & 28.8 & 5.76 & 615.168 \end{array}\right] \\ \hline \left[\begin{array}{ccc:c} 0 & -16.8 & -4.76 & -335.968 \end{array}\right] \end{array}$$

Substitusi pers baru ke pers. 3

$$\left[\begin{array}{ccc:c} 25 & 5 & 1 & 106.8 \\ 0 & -4.8 & -1.56 & -96.208 \\ 0 & -16.8 & -4.76 & -335.968 \end{array}\right]$$

Step 2

$$\left[\begin{array}{cccc} 25 & 5 & 1 & 106.8 \\ 0 & -4.8 & -1.56 & -96.208 \\ 0 & -16.8 & -4.76 & -335.968 \end{array}\right]$$

Bagi pers. 2 dg -4.8

Dan kalikan dg -16.8,

$$\frac{-16.8}{-4.8} = 3.5$$

$$\left[\begin{array}{cccc} 0 & -4.8 & -1.56 & -96.208 \end{array}\right] \times 3.5 = \left[\begin{array}{cccc} 0 & -16.8 & -5.46 & -336.728 \end{array}\right]$$

Kurangkan hasilnya dg pers. 3

$$\begin{array}{r} \left[\begin{array}{cccc} 0 & -16.8 & -4.76 & 335.968 \end{array}\right] \\ -\left[\begin{array}{cccc} 0 & -16.8 & -5.46 & -336.728 \end{array}\right] \\ \hline \left[\begin{array}{cccc} 0 & 0 & 0.7 & 0.76 \end{array}\right] \end{array}$$

Substitusi pers baru ke pers. 3

$$\left[\begin{array}{cccc} 25 & 5 & 1 & 106.8 \\ 0 & -4.8 & -1.56 & -96.208 \\ 0 & 0 & 0.7 & 0.76 \end{array}\right]$$

Substitusi balik

Substitusi balik

$$\left[\begin{array}{cccc} 25 & 5 & 1 & 106.8 \\ 0 & -4.8 & -1.56 & -96.2 \\ 0 & 0 & 0.7 & 0.7 \end{array}\right] \Rightarrow \left[\begin{array}{ccc} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{array}\right]\left[\begin{array}{c} a_1 \\ a_2 \\ a_3 \end{array}\right] = \left[\begin{array}{c} 106.8 \\ -96.208 \\ 0.76 \end{array}\right]$$

Penyelesaian untuk a_3

$$0.7a_3 = 0.76$$

$$a_3 = \frac{0.76}{0.7}$$

$$a_3 = 1.08571$$

Substitusi balik

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ -96.208 \\ 0.76 \end{bmatrix}$$

Penyelesaian untuk a_2

$$-4.8a_2 - 1.56a_3 = -96.208$$

$$a_2 = \frac{-96.208 + 1.56a_3}{-4.8}$$

$$a_2 = \frac{-96.208 + 1.56 \times 1.08571}{-4.8}$$

$$a_2 = 19.6905$$

Substitusi balik

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ -96.2 \\ 0.76 \end{bmatrix}$$

Peny untuk a_1

$$25a_1 + 5a_2 + a_3 = 106.8$$

$$a_1 = \frac{106.8 - 5a_2 - a_3}{25}$$

$$= \frac{106.8 - 5 \times 19.6905 - 1.08571}{25}$$

$$= 0.290472$$

Penyelesaian Gauss

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 0.290472 \\ 19.6905 \\ 1.08571 \end{bmatrix}$$

Contoh 1 – diperoleh hasil

Vektor dari a

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 0.290472 \\ 19.6905 \\ 1.08571 \end{bmatrix}$$

Kecepatan

$$v(t) = a_1 t^2 + a_2 t + a_3$$

$$= 0.290472t^2 + 19.6905t + 1.08571, \quad 5 \le t \le 12$$

$$v(6) = 0.290472(6)^2 + 19.6905(6) + 1.08571$$

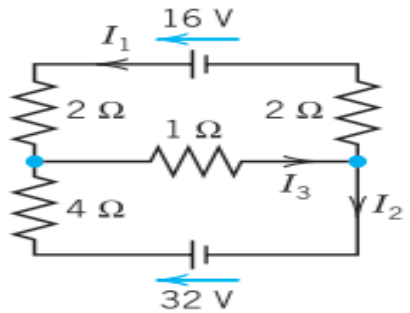
$$= 129.686 \text{ m/s}.$$

Tugas – dikumpulkan Jam 16.00 (6 Oktober 2020)

17–21 MODELS OF NETWORKS

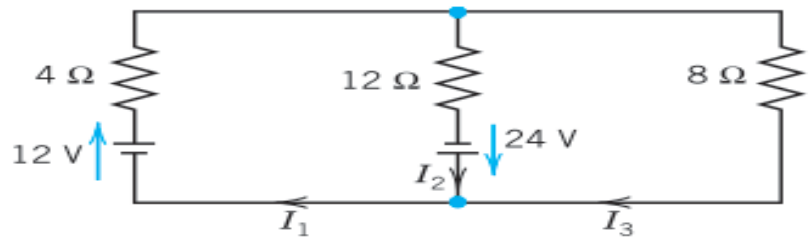
In Probs. 17–19, using Kirchhoff's laws (see Example 2) and showing the details, find the currents:

17.

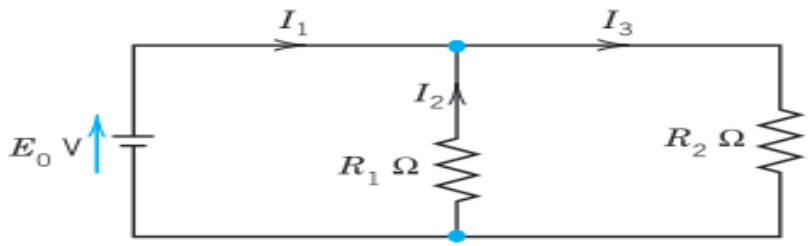


Pilih 2 dari 3 soal

18.

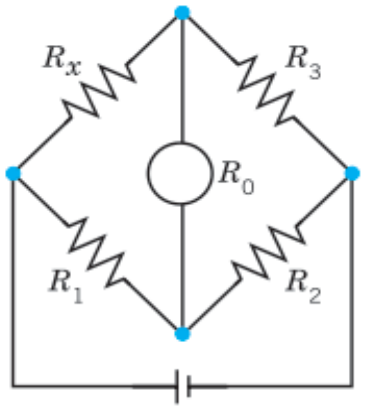


19.



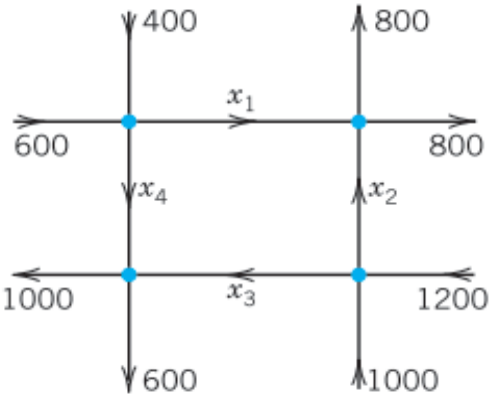
Dikerjakan semua

20. Wheatstone bridge. Show that if $R_x/R_3 = R_1/R_2$ in the figure, then $I = 0$. (R_0 is the resistance of the instrument by which I is measured.) This bridge is a method for determining R_x. R_1, R_2, R_3 are known. R_3 is variable. To get R_x, make $I = 0$ by varying R_3. Then calculate $R_x = R_3R_1/R_2$.



Wheatstone bridge

Problem 20



Net of one-way streets

Problem 21

terimakasih

Kerjakan Tugas – sesuai dg Panduan
(4 soal dari 5 soal)