PERSAMAAN ALJABAR LINIER

PENYELESAIAN DG MET. ELIMINASI GAUSS

MATEMATIKA REKAYASA 1

AULIA SITI AISJAH – TEKNIK FISIKA ITS

Pers. Aljabar Linier

Capaian Pembelajaran:

- Mampu menggunakan konsep iterasi Gauss Seidel untuk menyelesaian Persamaan Aljabar Linier
- Mampu membandingkan kelebihan dan kekurangan Met. Gauss Seidel dibandingkan dengan met. lain

Kajian:

1. **Metode Gauss Seidel**

$$\begin{array}{ccccccccc}
a_{11}x_1 & + & a_{12}x_2 & + & \cdots & + & a_{1n}x_n & = & b_1 \\
a_{21}x_1 & + & a_{22}x_2 & + & \cdots & + & a_{2n}x_n & = & b_2 \\
\vdots & & \vdots & & & & \vdots & & \vdots \\
a_{n1}x_1 & + & a_{n2}x_2 & + & \cdots & + & a_{nn}x_n & = & b_n
\end{array}$$

$$x_1 = \frac{1}{a_{11}}(b_1 - a_{12}x_2 - a_{13}x_3 - \cdots - a_{1n}x_n)$$

$$x_2 = \frac{1}{a_{22}}(b_2 - a_{21}x_1 - a_{23}x_3 - \cdots - a_{2n}x_n)$$

$$\vdots$$

$$x_n = \frac{1}{a_{nn}}(b_n - a_{n1}x_1 - a_{n2}x_2 - \cdots - a_{n,n-1}x_{n-1})$$

Contoh

$$\begin{aligned} 5x_1 - 2x_2 + 3x_3 &= -1 \\ -3x_1 + 9x_2 + x_3 &= 2 \\ 2x_1 - x_2 - 7x_3 &= 3 \end{aligned}$$

$$\begin{aligned} x_1 &= -\tfrac{1}{5} + \tfrac{2}{5}x_2 - \tfrac{3}{5}x_3 \\ x_2 &= \tfrac{2}{9} + \tfrac{3}{9}x_1 - \tfrac{1}{9}x_3 \\ x_3 &= -\tfrac{3}{7} + \tfrac{2}{7}x_1 - \tfrac{1}{7}x_2 . \end{aligned}$$

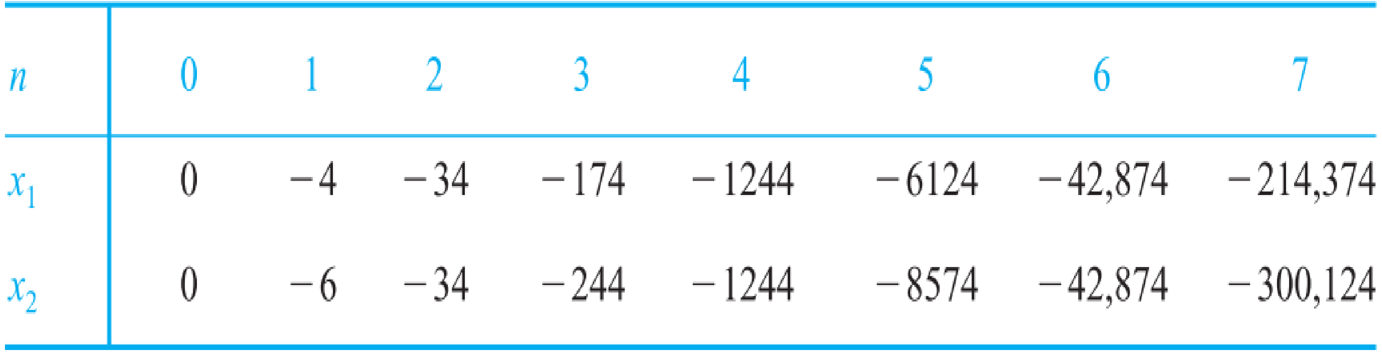
$$x_1 = 0, \qquad x_2 = 0, \qquad x_3 = 0$$

$$\begin{aligned} x_1 &= -\tfrac{1}{5} + \tfrac{2}{5}(0) - \tfrac{3}{5}(0) = -0.200 \\ x_2 &= \tfrac{2}{9} + \tfrac{3}{9}(0) - \tfrac{1}{9}(0) \approx 0.222 \\ x_3 &= -\tfrac{3}{7} + \tfrac{2}{7}(0) - \tfrac{1}{7}(0) \approx -0.429. \end{aligned}$$

n	0	1	2	3	4	5	6	7
x_1	0.000	−0.200	0.146	0.192	0.181	0.185	0.186	0.186
x_2	0.000	0.222	0.203	0.328	0.332	0.329	0.331	0.331
x_3	0.000	−0.429	−0.517	−0.416	−0.421	−0.424	−0.423	−0.423

Beberapa kasus – menghasilkan solusi divergen

$$\begin{aligned} x_1 - 5x_2 &= -4 \\ 7x_1 - x_2 &= 6, \end{aligned}$$



n	0	1	2	3	4	5	6	7
x_1	0	−4	−34	−174	−1244	−6124	−42,874	−214,374
x_2	0	−6	−34	−244	−1244	−8574	−42,874	−300,124

Dengan kondisi awal = 0

$$\begin{aligned} x_1 &= -4 + 5x_2 \\ x_2 &= -6 + 7x_1. \end{aligned}$$

$$\begin{aligned} x_1 &= -4 + 5(0) = -4 \\ x_2 &= -6 + 7(0) = -6 \end{aligned}$$

Dapat dilakukan dengan – perubahan posisi persamaan

Met. Gauss Seidel tidak selalu menghasilkan solusi → bisa terjadi solusi divergen

Contoh

$$\begin{aligned} x_1 - 5x_2 &= -4 \\ 7x_1 - x_2 &= 6 \end{aligned}$$

$$\Downarrow$$

$$\begin{aligned} 7x_1 - x_2 &= 6 \\ x_1 - 5x_2 &= -4. \end{aligned}$$

n	0	1	2	3	4	5
x_1	0.0000	0.8571	0.9959	0.9999	1.000	1.000
x_2	0.0000	0.9714	0.9992	1.000	1.000	1.000

$$\begin{aligned} x_1 &= \tfrac{6}{7} + \tfrac{1}{7}x_2 \\ x_2 &= \tfrac{4}{5} + \tfrac{1}{5}x_1 \end{aligned}$$

Selaikan Tugas – NRP Ganjil → No 23, NRP Genap → No 24

23.

$$\begin{aligned}
4x_1 + x_2 - x_3 &= 3\\
x_1 + 6x_2 - 2x_3 + x_4 - x_5 &= -6\\
x_2 + 5x_3 - x_5 + x_6 &= -5\\
2x_2 + 5x_4 - x_5 - x_7 - x_8 &= 0\\
-x_3 - x_4 + 6x_5 - x_6 - x_8 &= 12\\
-x_3 - x_5 + 5x_6 &= -12\\
-x_4 + 4x_7 - x_8 &= -2\\
-x_4 - x_5 - x_7 + 5x_8 &= 2
\end{aligned}$$

24.

$$\begin{aligned}
4x_1 - x_2 - x_3 &= 18\\
-x_1 + 4x_2 - x_3 - x_4 &= 18\\
-x_2 + 4x_3 - x_4 - x_5 &= 4\\
-x_3 + 4x_4 - x_5 - x_6 &= 4\\
-x_4 + 4x_5 - x_6 - x_7 &= 26\\
-x_5 + 4x_6 - x_7 - x_8 &= 16\\
-x_6 + 4x_7 - x_8 &= 10\\
-x_7 + 4x_8 &= 32
\end{aligned}$$

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Terimakasih