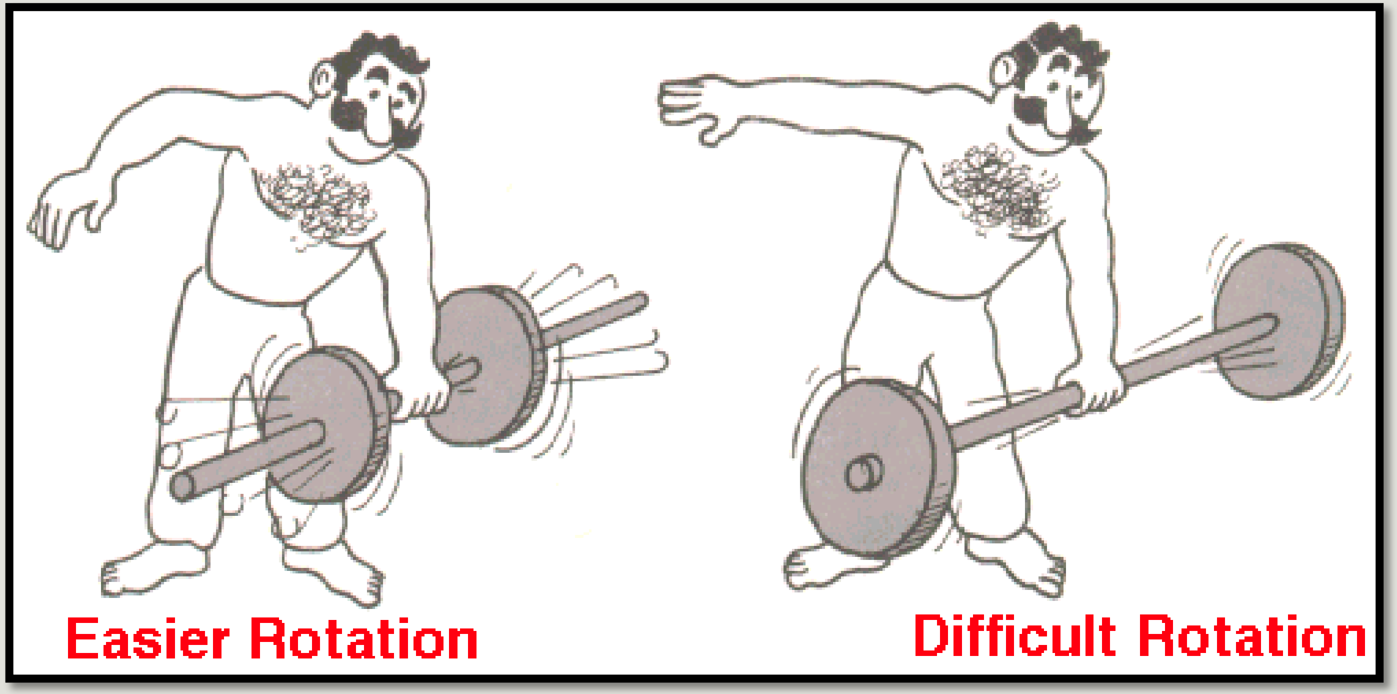
MOMENT OF INERTIA

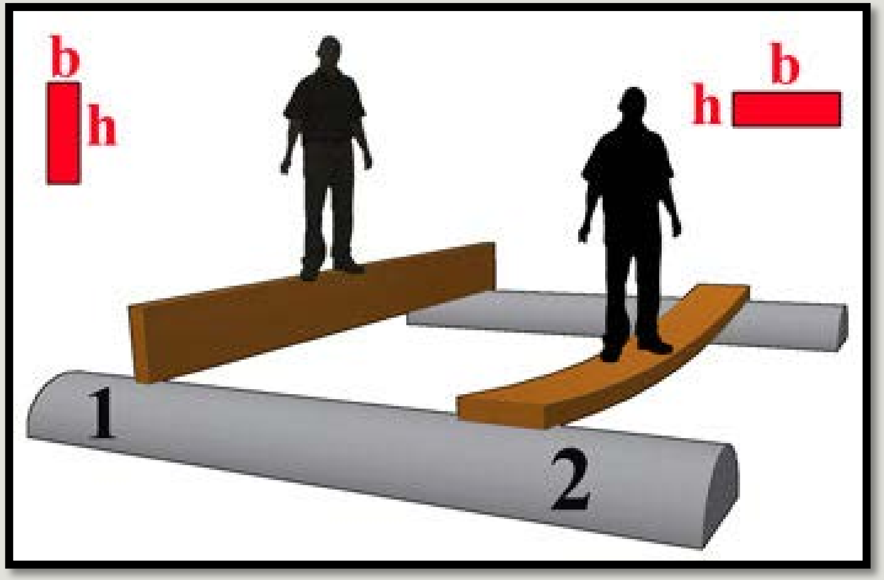
ENGINEERING MECHANICS

SUNARDI TJANDRA – MANUFACTURING ENGINEERING UBAYA

DEFINITION

- It is a measure of an object's resistance to changes to its rotation.
- Also defined as the capacity of a cross-section to resist bending.
- It must be specified with respect to a chosen axis of rotation.
- It is usually quantified in ***m⁴*** or ***kgm²***



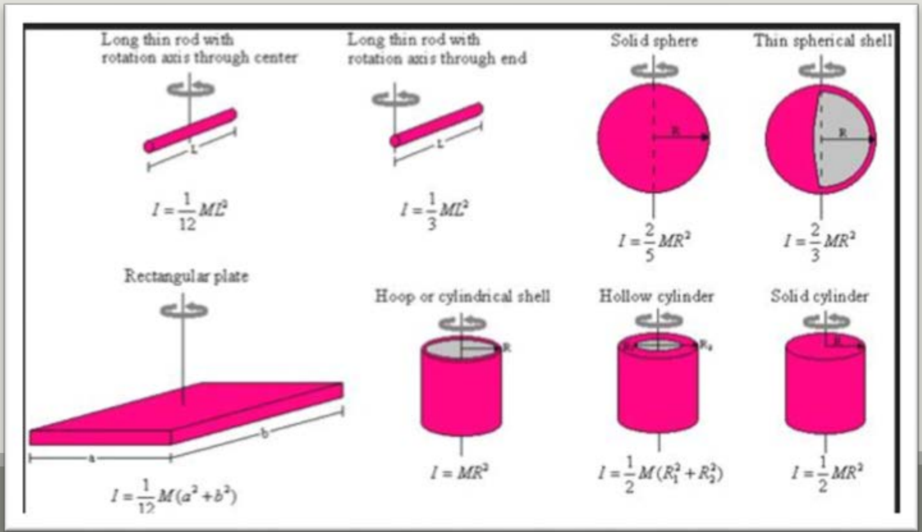


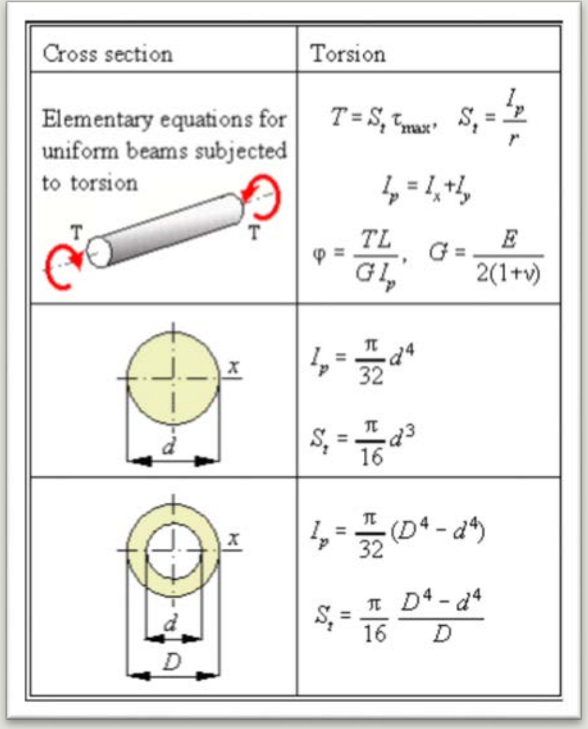
JENIS MOMEN INERSIA

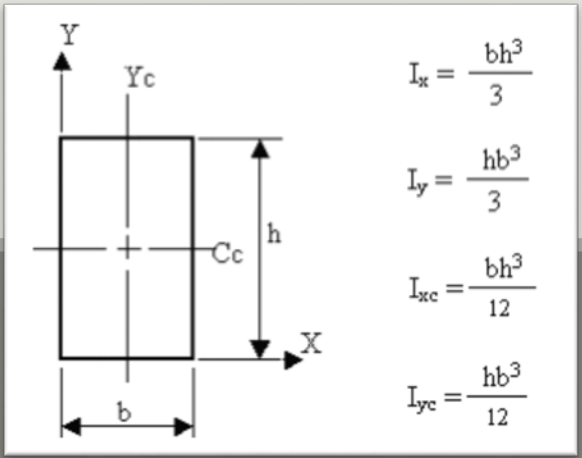
Momen Inersia Bidang/Penampang

Momen Inersia Massa

Momen Inersia Polar







FAKTOR PENENTU

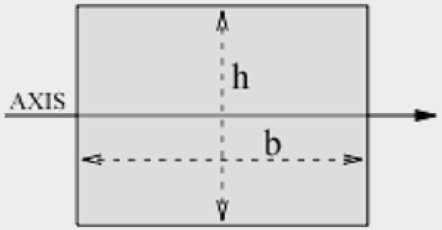
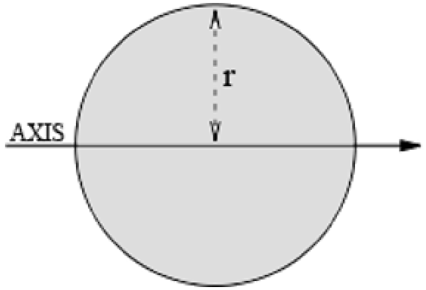
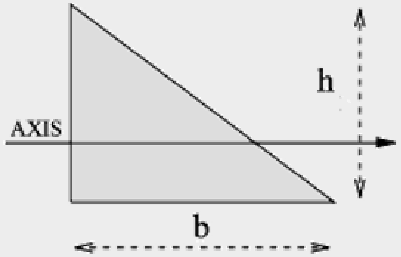
Momen Inersia Bidang/Penampang

- Luas Area
- Pusat Massa
- Jarak Sumbu Putar

Momen Inersia Massa & Polar

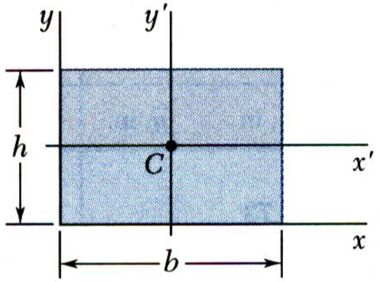
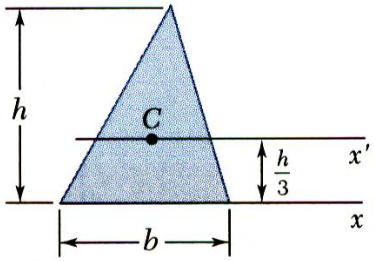
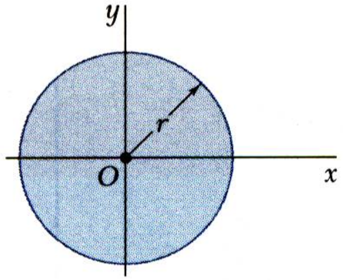
- Massa benda & Pola Distribusinya
- Sumbu Rotasi
- Jarak posisi rotasi

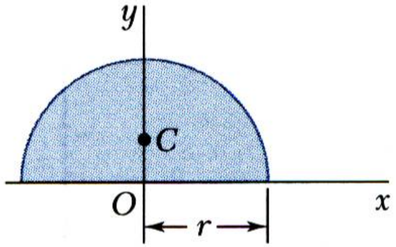
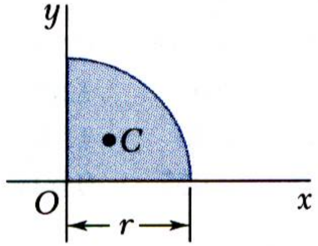
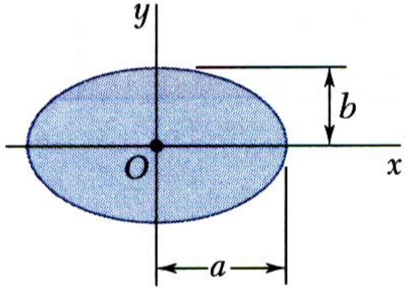
MOMEN INERSIA BIDANG

Shape	Ic (Centroidal 2nd Moment of Area)	Centroid (Centre of Area)	Area
	**Rectangle** Bending about centroid (centre). $I_c = \frac{bh^3}{12}$ *b = breadth, h = height*	At centre	A = b*h
	**Circle** Bending about centroid (centre). $I_c = \frac{\pi d^4}{64}$ *r = radius*	At centre	A = Π*r^2
	**Triangle** Bending about centroid. $I_0 = \frac{bh^3}{36}$ *b = breadth, h = height*	Xc = h/3 Yc = b/3	A = 0.5*b*h

MOMEN INERSIA BIDANG

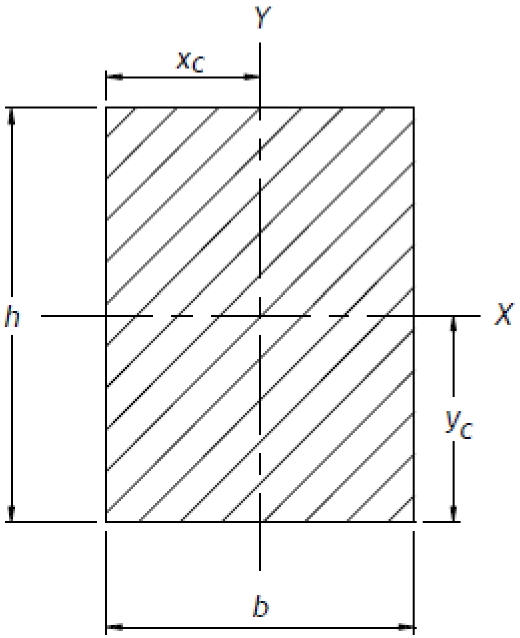
BENTUK-BENTUK GEOMETRI DASAR

Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$

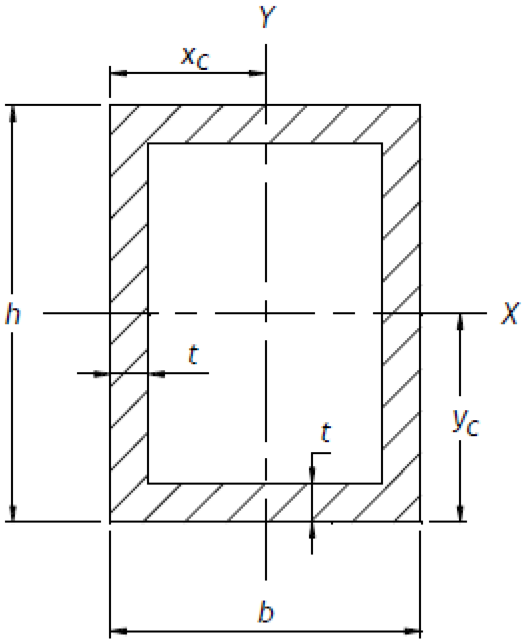
Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$
Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$

MOMEN INERSIA BIDANG

BENTUK-BENTUK GEOMETRI DASAR



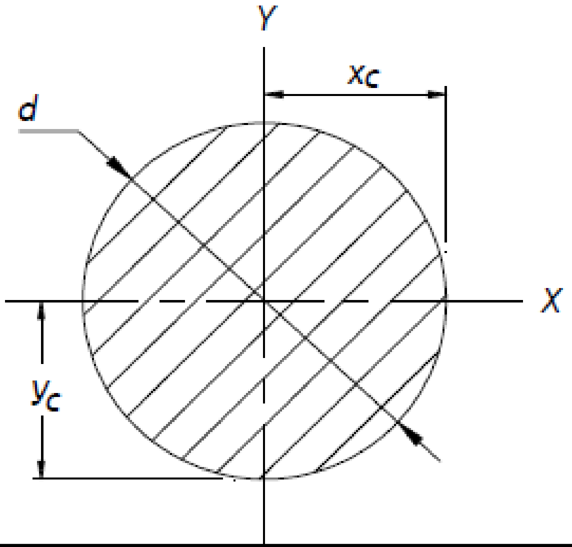
$$x_c = \tfrac{1}{2}h$$
$$y_c = \tfrac{1}{2}b$$
$$A = bh$$
$$I_x = \tfrac{1}{12}bh^3$$
$$I_y = \tfrac{1}{12}hb^3$$



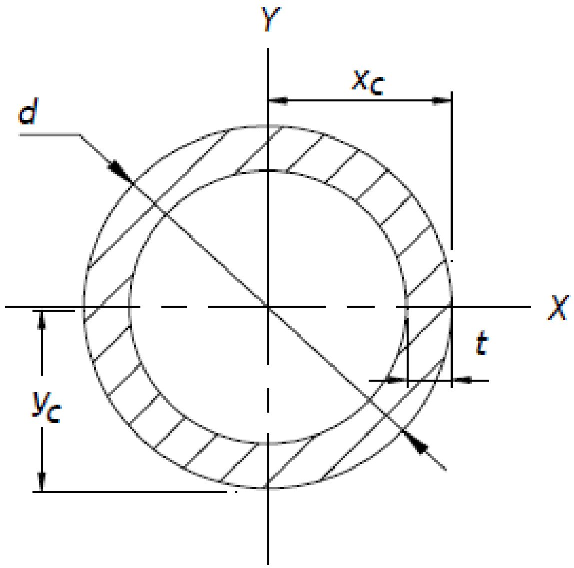
$$x_c = \tfrac{1}{2}h$$
$$y_c = \tfrac{1}{2}b$$
$$A = 2t(b+h) - 4t^2$$
$$I_x = \tfrac{1}{12}\left[bh^3 - (b-2t)(h-2t)^3\right]$$
$$I_y = \tfrac{1}{12}\left[hb^3 - (h-2t)(b-2t)^3\right]$$

MOMEN INERSIA BIDANG

BENTUK-BENTUK GEOMETRI DASAR



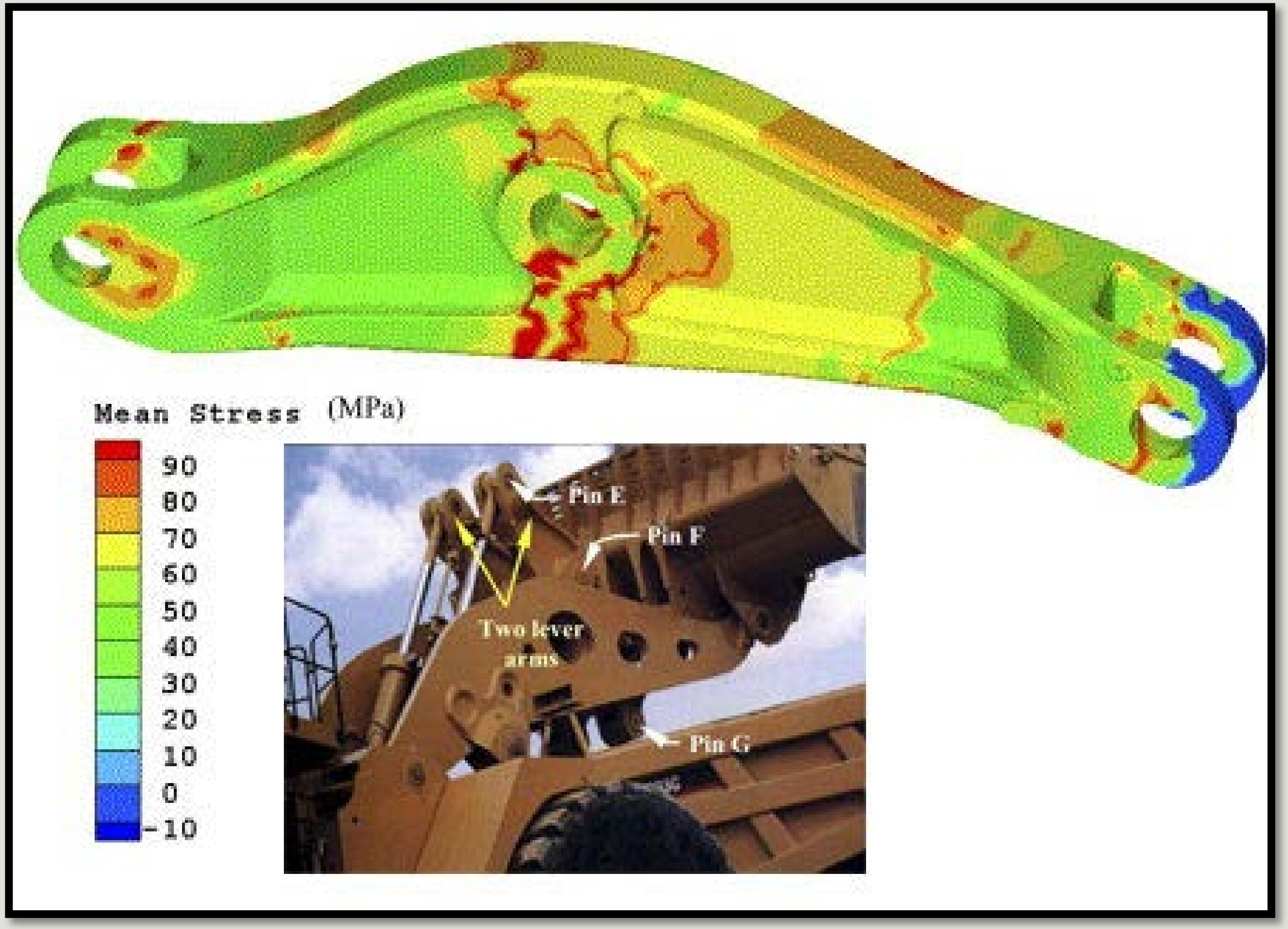
$$x_c = \tfrac{1}{2}d$$
$$y_c = \tfrac{1}{2}d$$
$$A = \tfrac{\pi}{4}d^2$$
$$I_x = \tfrac{\pi}{64}d^4$$
$$I_y = \tfrac{\pi}{64}d^4$$

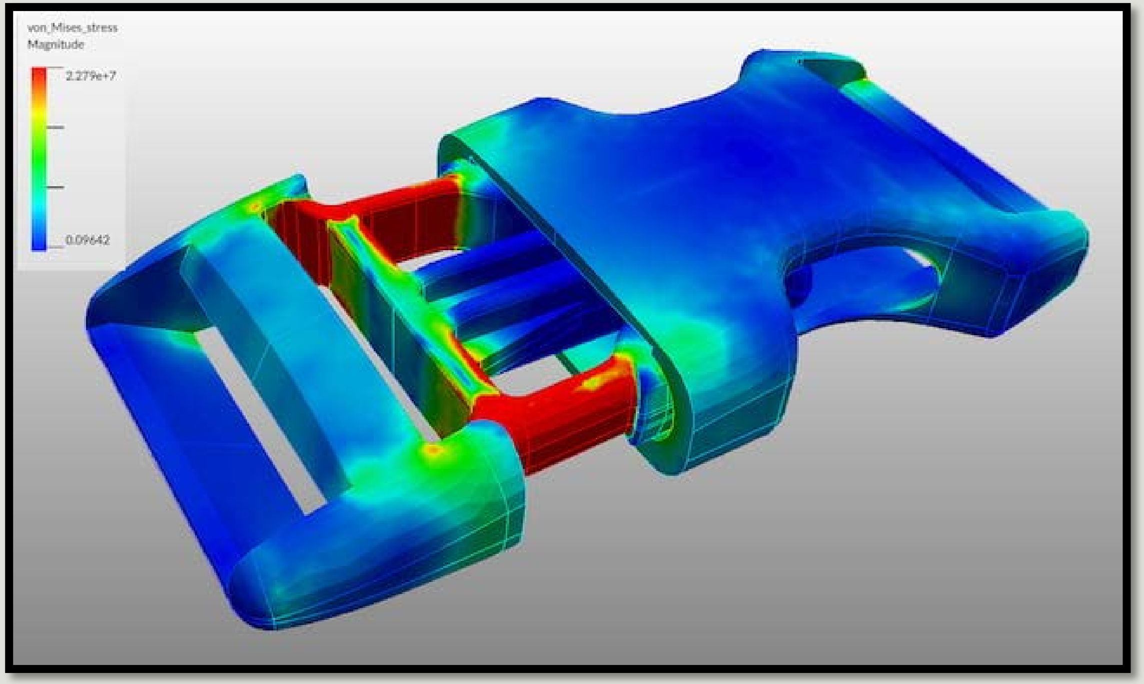


$$x_c = \tfrac{1}{2}d$$
$$y_c = \tfrac{1}{2}d$$
$$A = \pi t\left(d - t\right)$$
$$I_{x,y} = \tfrac{\pi}{64}\left(d^4 - \left(d - 2t\right)^4\right)$$

MOMEN INERSIA BIDANG

BENTUK-BENTUK GEOMETRI DASAR





MOMEN INERSIA BIDANG

CONTOH APLIKASI

TEOREMA SUMBU SEJAJAR

Momen Inersia bidang terhadap sumbu tertentu dapat dihitung berdasarkan momen inersia bidang **terhadap sumbu titik berat yang sejajar.**

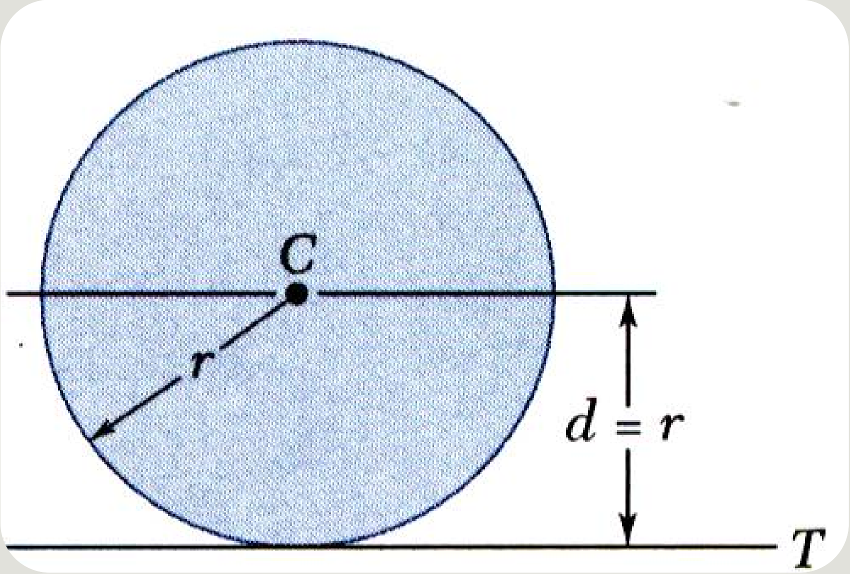
$$I = \bar{I} + Ad^2$$

dimana:

- **I** = momen inersia bidang terhadap sumbu tertentu
- **$\bar{I}$** = momen inersia bidang terhadap pusat massanya sendiri
- **A** = luas penampang bidang
- **d** = jarak antara pusat massa bidang ke sumbu tertentu

CONTOH 1

Tentukan momen inersia bidang dari lingkaran berikut ini terhadap sumbu T



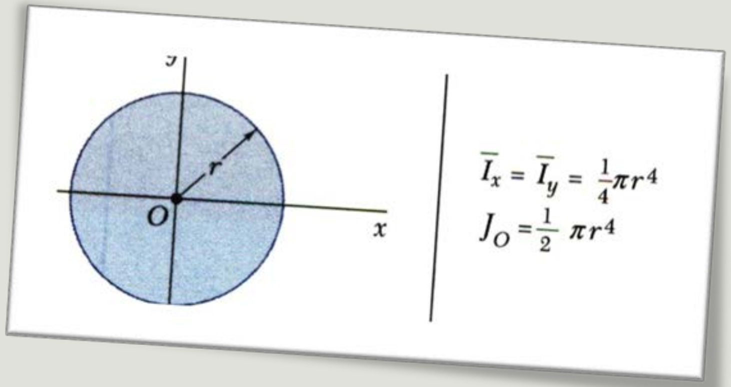
$$I = \bar{I} + \mathrm{A}d^2$$

$$A = \pi r^2$$

$$\bar{I} = \frac{1}{4}\pi r^4$$

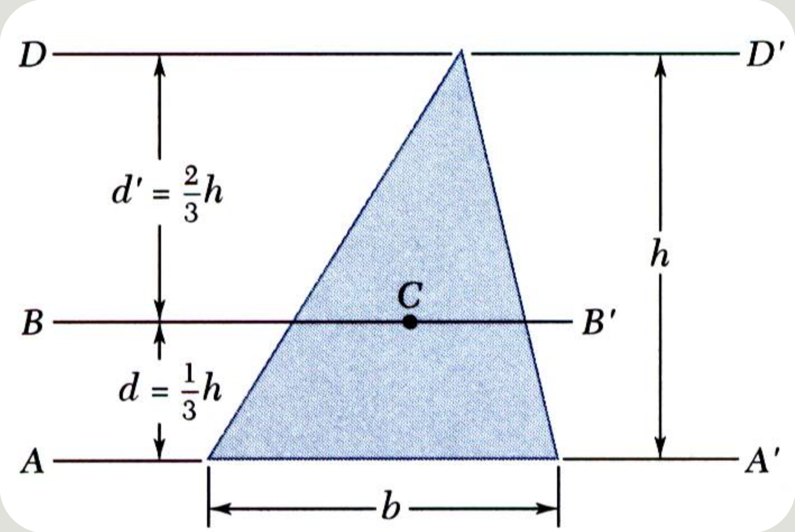
$$I = \frac{1}{4}\pi r^4 + (\pi r^2)r^2$$

$$= \frac{5}{4}\pi r^4$$



CONTOH 2

Tentukan momen inersia bidang dari segitiga berikut ini terhadap pusat massanya

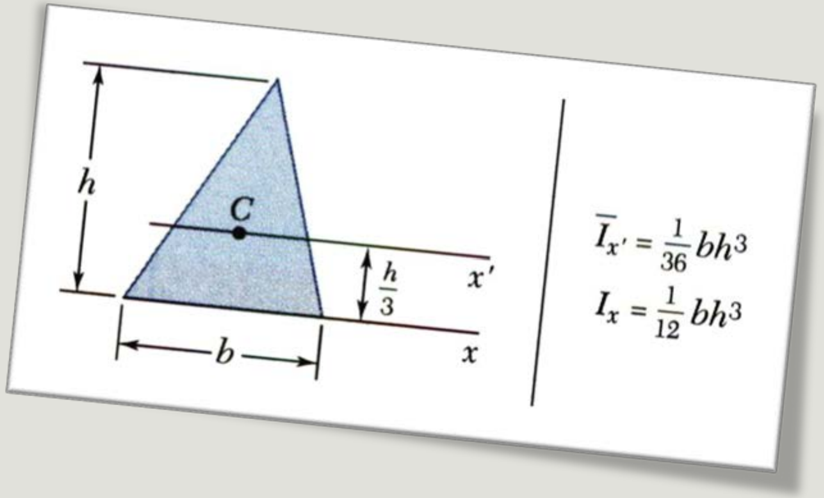


$$I_{AA'} = \bar{I}_{BB'} + Ad^2$$

$$\bar{I}_{BB'} = I_{AA'} - Ad^2$$

$$A = \frac{1}{2}bh$$

$$I_{AA'} = \frac{1}{12}bh^3$$

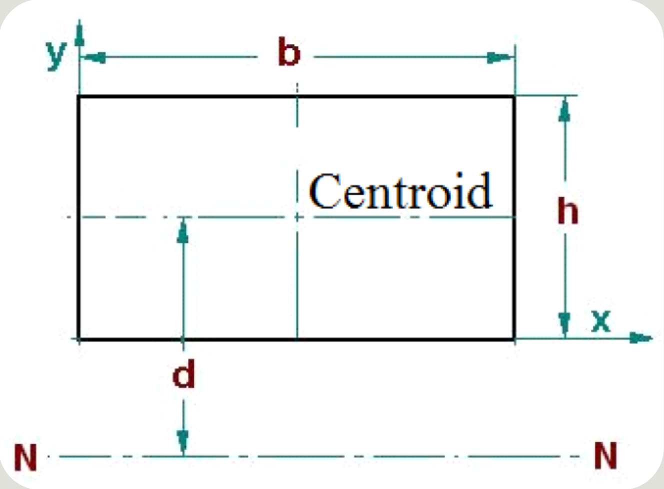


$$I = \frac{1}{12}bh^3 - (\frac{1}{2}bh)(\frac{1}{3}h)^2$$

$$= \frac{1}{36}bh^3$$

CONTOH 3

Hitung momen inersia bidang berikut terhadap sumbu N-N, dimana **b = 18mm, h = 4,9mm, d = 6,2mm**



$$\boldsymbol{I} = \bar{\boldsymbol{I}} + \mathbf{A}\boldsymbol{d}^{\mathbf{2}}$$

$$\bar{I} = \frac{1}{12} bh^3$$

$$= \frac{1}{12} . 18 . 4{,}9^3$$

$$= 176{,}4735\ mm^4$$

$$A = bh$$

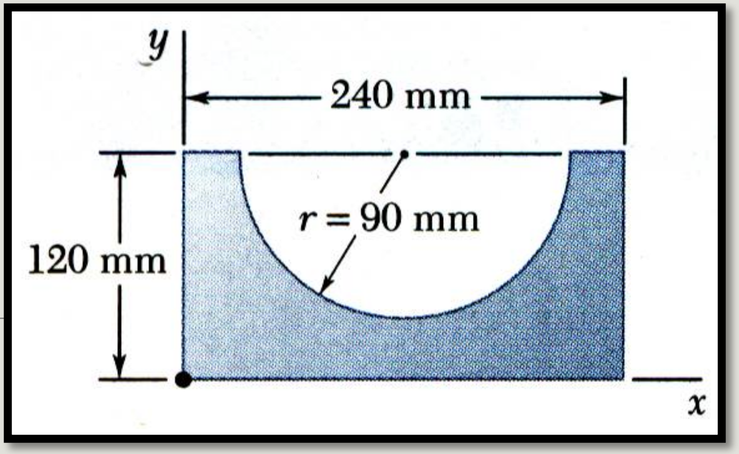
$$= 18 . 4{,}9$$

$$= 88{,}2\ mm^2$$

$$I = 176{,}4735 + 88{,}2 . 6{,}2^2$$

$$= 3566{,}9\ mm^4$$

CONTOH 4

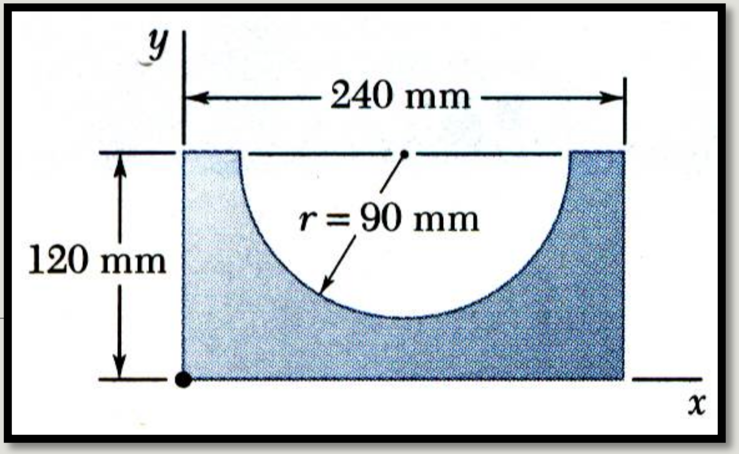


Hitung momen inersia bidang berikut terhadap **sumbu X**

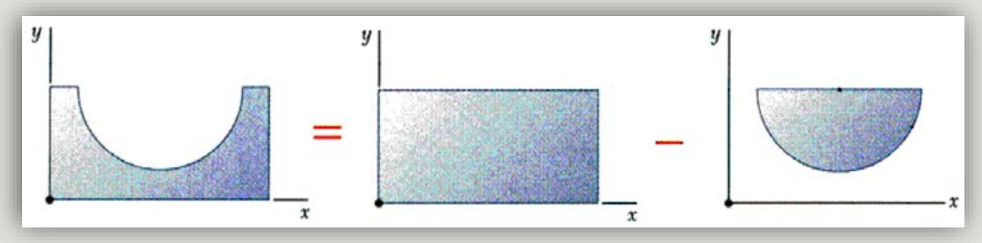
SOLUSI:

1. Membagi bidang menjadi bidang-bidang dasar.
2. Jika diperlukan, menghitung titik berat setiap bidang dasar ($\bar{x}_i$)
3. Menghitung titik berat bidang utama
4. Menghitung luas area (A_i) setiap bidang dasar.
5. Jika diperlukan, menghitung momen inersia setiap bidang dasar terhadap titik beratnya ($\bar{I}_i$).
6. Menghitung jarak (d_i) dari titik berat setiap bidang ke titik berat bidang utama.
7. Menghitung momen inersia bidang utama (I_n) menggunaan teori sumbu sejajar.

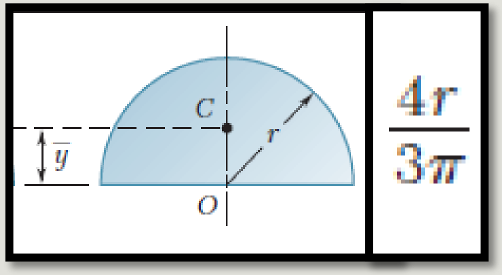
CONTOH 4 (lanj.)



Membagi bidang menjadi bidang-bidang dasar.



Menghitung titik berat bidang setengah lingkaran ($\bar{x}_i$)

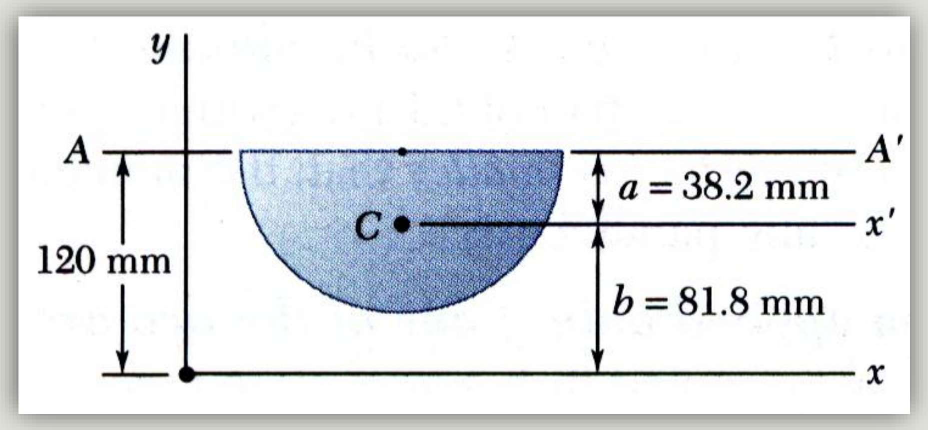


$$a = \frac{4r}{3\pi} = \frac{(4)(90)}{3\pi} = 38.2\ \text{mm}$$

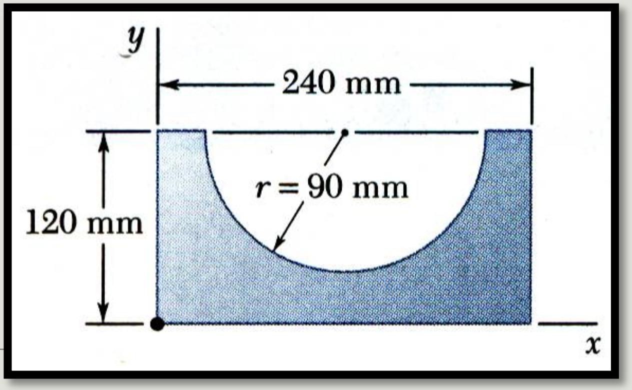
$$\text{b} = 120 - \text{a} = 81.8\ \text{mm}$$

$$A = \tfrac{1}{2}\pi r^2 = \tfrac{1}{2}\pi(90)^2$$

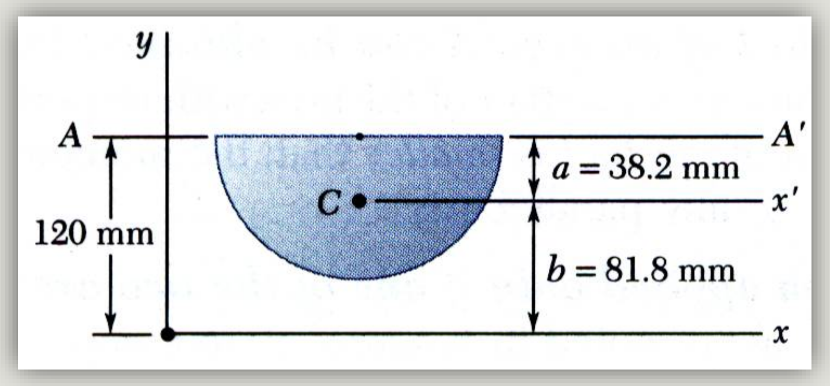
$$= 12.72 \times 10^3\ \text{mm}^2$$



CONTOH 4 (lanj.)



Menghitung momen inersia bidang-bidang dasar terhadap sumbu yang diinginkan.



Momen inersia bidang setengah lingkaran terhadap ***sumbu A-A'***:

$$I_{AA'} = \tfrac{1}{8}\pi r^4 = \tfrac{1}{8}\pi(90)^4 = 25.76\times10^6\,\text{mm}^4$$

Momen inersia bidang setengah lingkaran terhadap **titik beratnya**:

$$\bar{I}_{x'} = I_{AA'} - Aa^2 = \left(25.76\times10^6\right)\left(12.72\times10^3\right)$$

$$= 7.20\times10^6\,\text{mm}^4$$

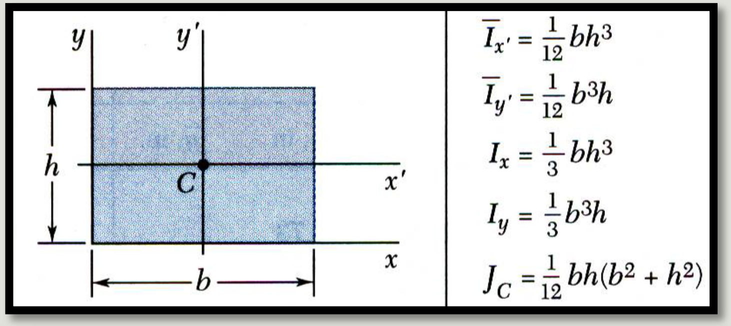
Momen inersia bidang setengah lingkaran terhadap ***sumbu x***:

$$I_x = \bar{I}_{x'} + Ab^2 = 7.20\times10^6 + \left(12.72\times10^3\right)(81.8)^2$$

$$= 92.3\times10^6\,\text{mm}^4$$

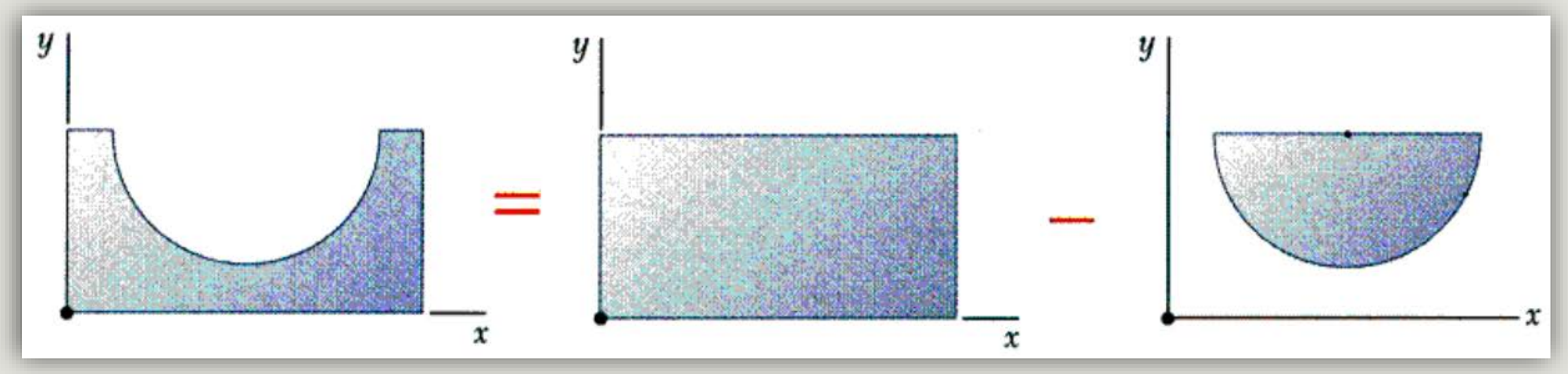
CONTOH 4 (lanj.)

Menghitung momen inersia bidang-bidang dasar terhadap sumbu yang diinginkan.



Momen inersia bidang persegi panjang terhadap ***sumbu x***:

$$I_x = \tfrac{1}{3}bh^3 = \tfrac{1}{3}(240)(120) = 138.2 \times 10^6 \,\mathrm{mm}^4$$



$$I_x = 45.9 \times 10^6 \,\mathrm{mm}^4$$

$$I_x \quad = \quad 138.2 \times 10^6 \,\mathrm{mm}^4 \quad - \quad 92.3 \times 10^6 \,\mathrm{mm}^4$$

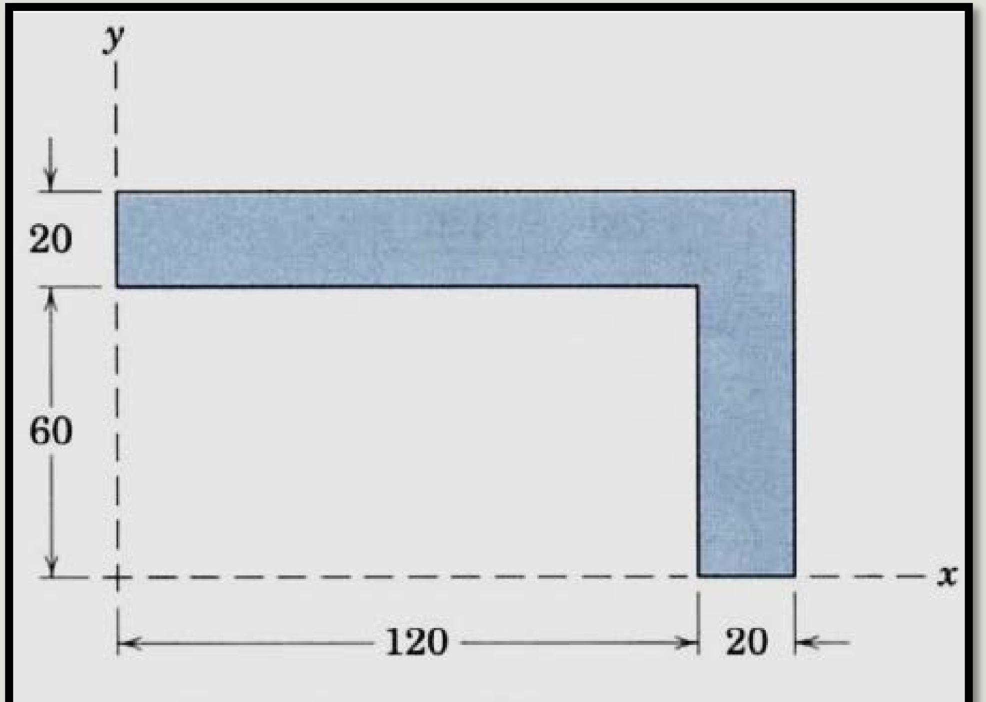
LATIHAN SOAL

MOMEN INERSIA BIDANG

SUNARDI TJANDRA – MANUFACTURING ENGINEERING UBAYA

LATIHAN 1

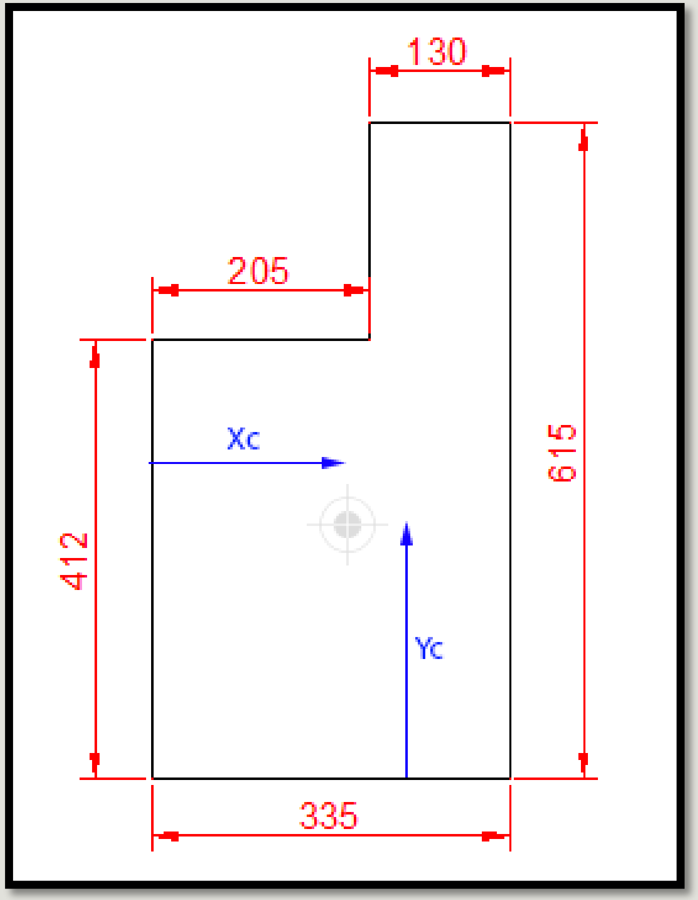
Hitung momen inersia bidang berikut terhadap ***sumbu x***



Dimensions in millimeters

LATIHAN 2

Hitung momen inersia bidang berikut terhadap **titik beratnya**



MOMEN INERSIA MASSA

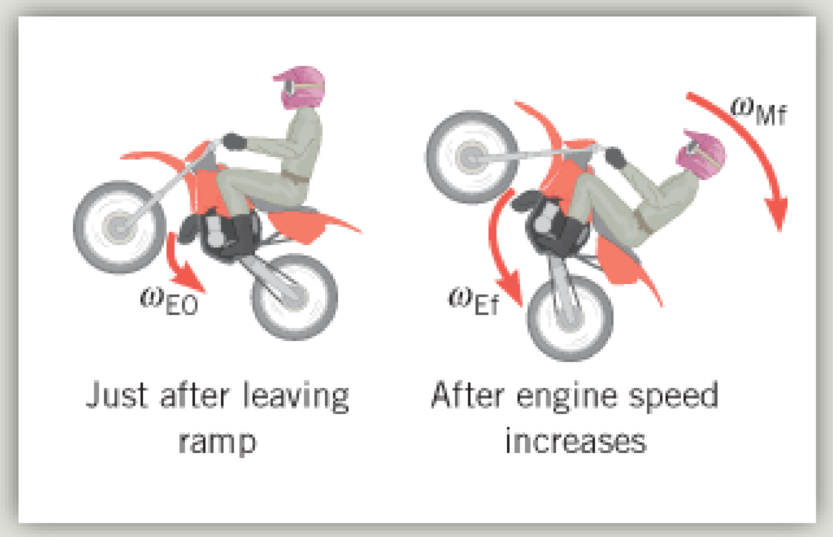
DEFINISI

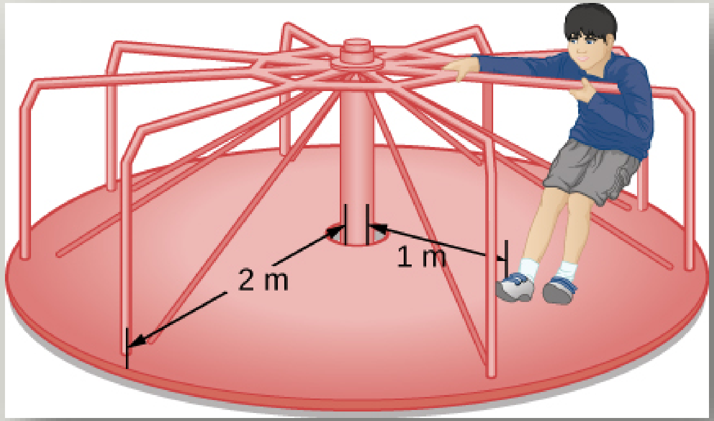
$$I = \int r^2 dm$$

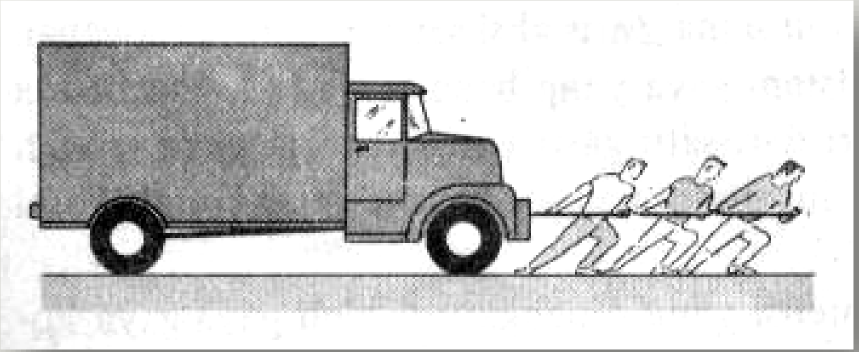
Benda yang sedang bergerak rotasi akan memiliki kecenderungan untuk tetap mempertahankan gerak rotasinya.

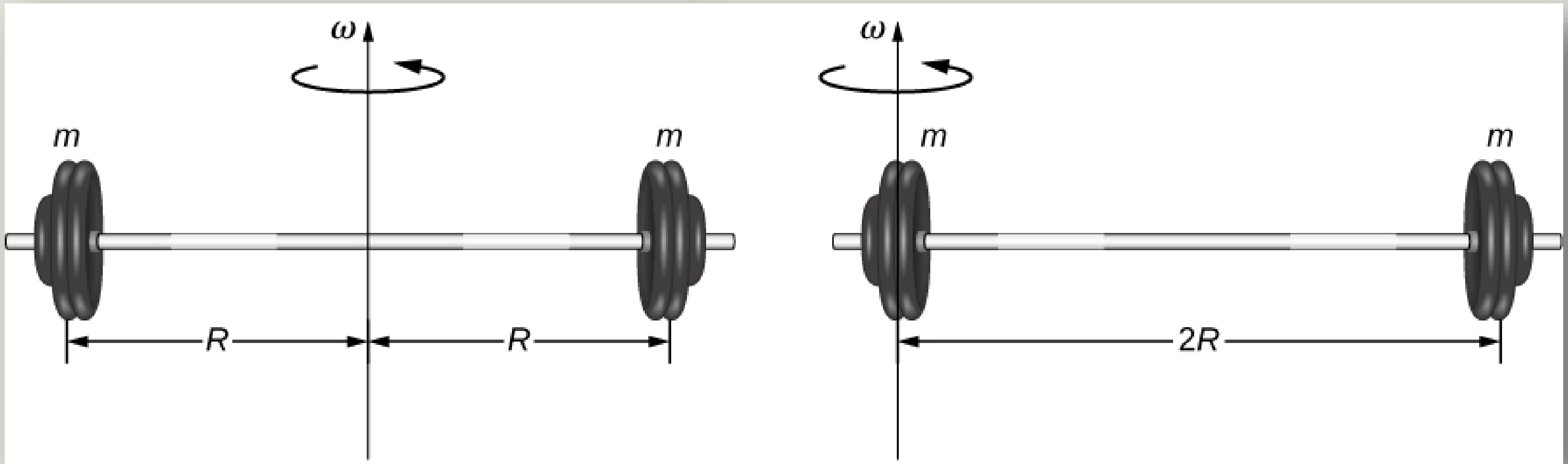
Kecenderungan benda untuk mempertahankan gerak rotasinya

Perkalian massa dengan jarak kuadrat dari sumbu



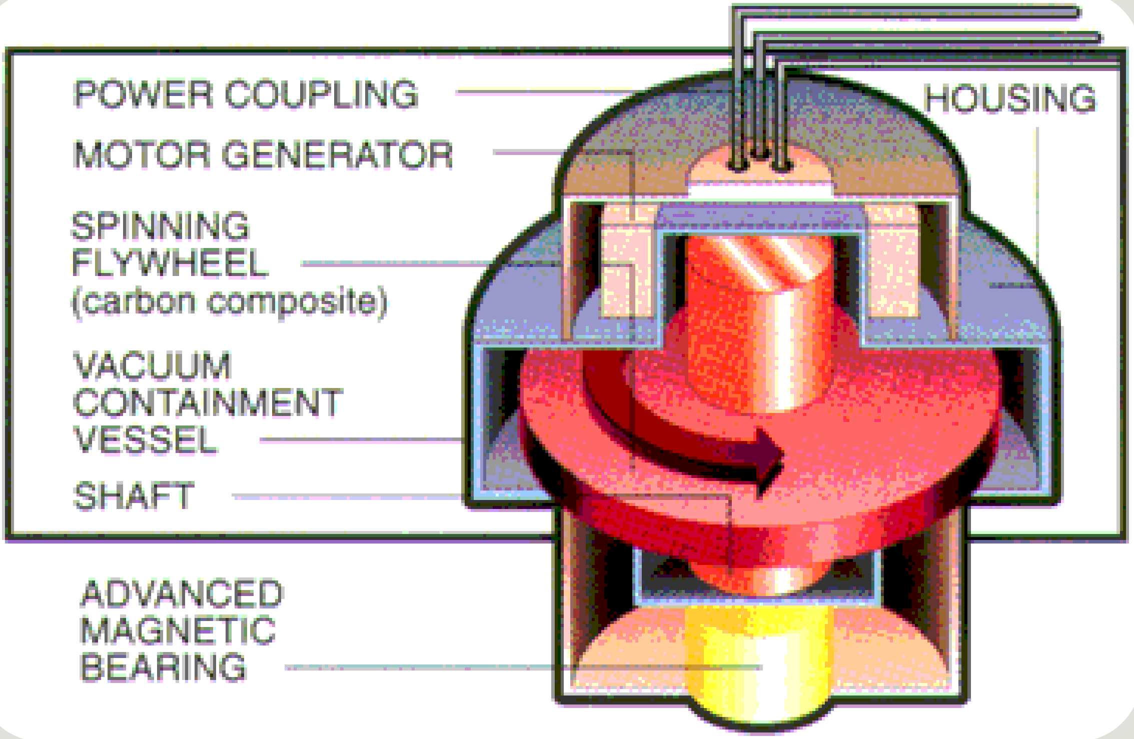



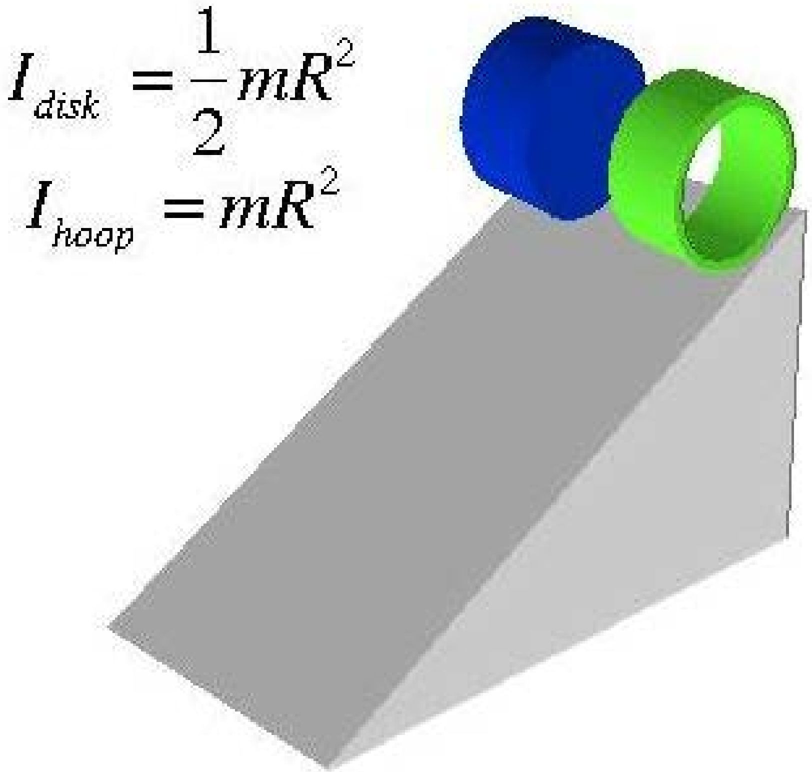




MOMEN INERSIA MASSA

CONTOH APLIKASI





MOMEN INERSIA MASSA

CONTOH APLIKASI

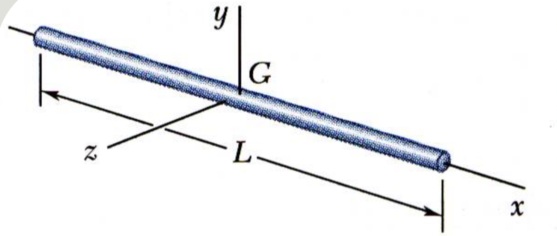
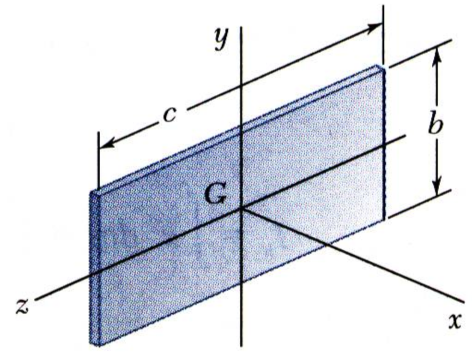
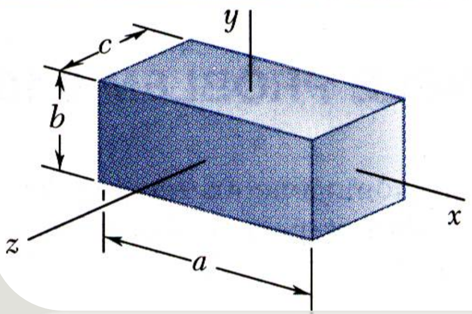
TEOREMA SUMBU SEJAJAR

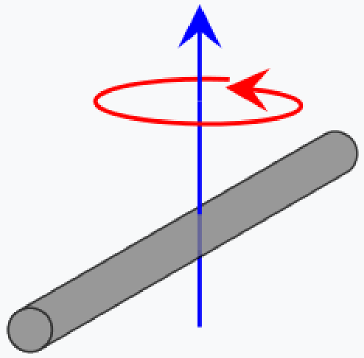
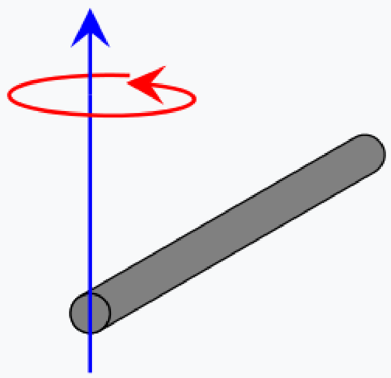
Momen inersia massa terhadap sumbu tertentu dapat dihitung berdasarkan momen inersia massa terhadap **sumbu titik berat yang sejajar**

$$I_x = \bar{I}_{x`} + m(\bar{y}^2 + \bar{z}^2)$$

$$I_y = \bar{I}_{y`} + m(\bar{x}^2 + \bar{z}^2)$$

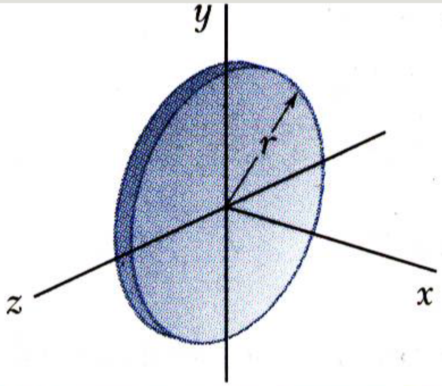
$$I_z = \bar{I}_{z`} + m(\bar{x}^2 + \bar{y}^2)$$

	$I_y = I_z = \frac{1}{12} mL^2$
	$I_x = \frac{1}{12} m(b^2 + c^2)$ $I_y = \frac{1}{12} mc^2$ $I_z = \frac{1}{12} mb^2$
	$I_x = \frac{1}{12} m(b^2 + c^2)$ $I_y = \frac{1}{12} m(c^2 + a^2)$ $I_z = \frac{1}{12} m(a^2 + b^2)$

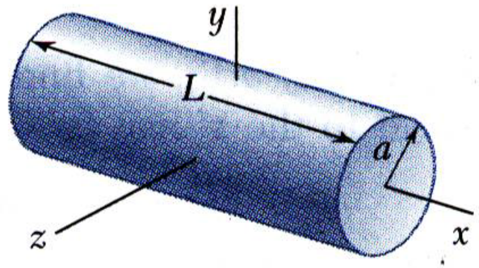
	$I_{center} = \frac{1}{12} mL^2$ [1]
	$I_{end} = \frac{1}{3} mL^2$ [1]

MOMEN INERSIA MASSA

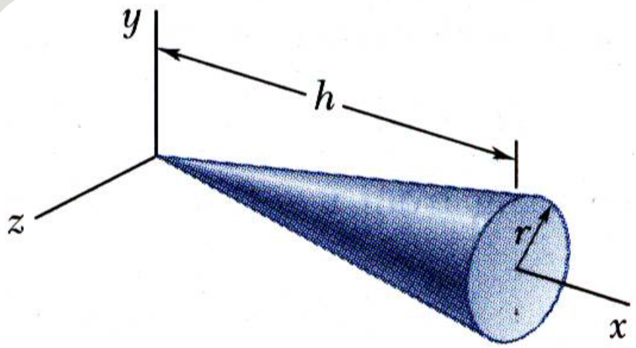
BENTUK-BENTUK GEOMETRI DASAR



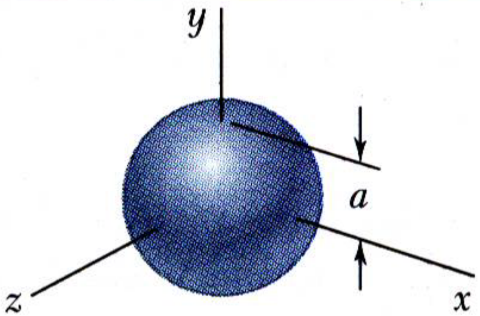
$$I_x = \frac{1}{2} m r^2$$
$$I_y = I_z = \frac{1}{4} m r^2$$



$$I_x = \frac{1}{2} m a^2$$
$$I_y = I_z = \frac{1}{12} m(3a^2 + L^2)$$



$$I_x = \frac{3}{10} m a^2$$
$$I_y = I_z = \frac{3}{5} m(\frac{1}{4} a^2 + h^2)$$



$$I_x = I_y = I_z = \frac{2}{5} m a^2$$

MOMEN INERSIA MASSA

BENTUK-BENTUK GEOMETRI DASAR

CONTOH

Berapa torsi yang dibutuhkan oleh kincir untuk bisa berputar dari keadaan diam sehingga menjadi 60 rpm jika kincir yang berputar mempunyai momen inersia sebesar 10 kg.m^2?

THANK YOU

END OF CHAPTER

SUNARDI TJANDRA – MANUFACTURING ENGINEERING UBAYA