

PERCENTS

Percent means “per hundred.” Writing a number as a percent is a way of comparing the number with 100. For example: $42\% = \frac{42}{100}$

Percents are really fractions (or ratios) with a denominator of 100. Any percent may be changed to an equivalent fraction by dropping the percent symbol and writing the number over 100. Usually it is best to put this fraction in simplest terms.

CHANGING PERCENTS TO DECIMALS

RULE: *To change a percent to a decimal, drop the % symbol and move the decimal point two places to the left.*

Examples: $25\% = 0.25$ $75\% = 0.75$ $6.8\% = 0.068$ $0.63\% = 0.0063$

CHANGING DECIMALS TO PERCENTS

RULE: *To change a decimal to a percent, move the decimal point two places to the right and use the % symbol.*

Examples: $0.27 = 27\%$ $4.89 = 489\%$ $0.2 = 20\%$ $25 = 2500\%$

CHANGING PERCENTS TO FRACTIONS

RULE: *To change a percent to a fraction, drop the % symbol and write the original number over 100. Simplify the fraction to lowest terms.*

Examples: $62\% = \frac{62}{100} = \frac{31}{50}$

$$4.5\% = \frac{4.5}{100} = \frac{4.5 \times 10}{100 \times 10} = \frac{45}{1000} = \frac{9}{200}$$

To create a whole number in the numerator, multiply the numerator and denominator by 10. Simplify.

$$32\frac{1}{2}\% = \frac{32\frac{1}{2}}{100} = \frac{\frac{65}{2}}{100} = \frac{65}{2} \times \frac{1}{100} = \frac{65}{200} = \frac{13}{40}$$

Writing $32\frac{1}{2}\%$ over 100 produces a complex fraction, so we change $32\frac{1}{2}$ to an improper fraction and simplify.

CHANGING FRACTIONS TO PERCENTS

RULE: *To change a fraction to a percent, change the fraction to a decimal and then change the decimal to a percent.*

Examples:

$$\frac{7}{10} = 0.7 = 70\%$$

Change $\frac{7}{10}$ to a decimal by dividing 7 by 10. Then change the resulting decimal 0.7 to a percent by moving the decimal point two places to the right and use the % symbol.

$$\frac{3}{8} = 0.375 = 37.5\%$$

Change $\frac{3}{8}$ to a decimal by dividing 3 by 8. Then change the decimal to a percent by moving the decimal point two places to the right and use the % symbol. Division equals 0.375 which becomes 37.5%.

BASIC PERCENT WORD PROBLEMS

There are three types of word problems associated with percents:

Type A: What number is 15% of 63?

Type B: What percent of 42 is 21?

Type C: 25 is 40% of what number?

The method we use to solve all three types of problems involves translating the sentences into equations and then solving the equations.

The following translations are used to equations:

<i>English</i>	<i>Mathematics</i>
is	=
of	x (multiply)
a number	n
what percent	n
what number	n

The word *is* always translates to an = sign, the word *of* almost always means multiply, and the number we are looking for can be represented with a letter, such as n or x .

Example 1 (Type A): What number is 15% of 63?

We translate the sentence into an equation as follows:

What *number is 15% of 63?*

↓ ↓ ↓ ↓ ↓

$n = 0.15 \cdot 63$

To do arithmetic with percents, we have to change percents to decimals.

Solving the equation, we have:

$$n = 0.15 \cdot 63$$

$$n = 9.45$$

15% of 63 is 9.45

Example 2 (Type B): What percent of 42 is 21?

We translate the sentence into an equation as follows:

What *percent of 42 is 21?*

↓ ↓ ↓ ↓ ↓

$n \cdot 42 = 21$

We solve for n by dividing both sides by 42.

$$\frac{n \cdot \cancel{42}}{\cancel{42}} = \frac{21}{42}$$

$$n = \frac{21}{42}$$

$$n = 0.50$$

Since the original problem asked for a percent, we change 0.50 to a percent.

$$n = 0.50 = 50\%$$

21 is 50% of 42

Example 3 (Type C): 25 is 40% of what number?

We translate the sentence into an equation as follows:

25 is 40% of what number?

$$\begin{array}{ccccccc} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ 25 & = & 0.40 & \cdot & n & & \end{array}$$

$$\frac{25}{0.40} = \frac{0.40 \cdot n}{0.40}$$

$$\frac{25}{0.40} = n$$

$$62.5 = n$$

25 is 40% of 62.5

As you can see, all three types of percent problems are solved in a similar manner. We write *is* as =, *of* as \cdot , and *what number* as n . The resulting equation is then solved to obtain the answer.

Percent problems may also be set up in a proportion format. $\frac{\text{percent}}{100} = \frac{\text{amount}}{\text{base}}$

To solve a proportion use the cross-multiplication method to establish an equation statement. In this case: percent \cdot base = amount \cdot 100

Using **Example 1**, the problem will be worked as follows:

What number is 15% of 63?

$$\frac{15}{100} = \frac{n}{63}$$

$$100 \cdot n = 15 \cdot 63$$

$$100n = 945$$

$$\frac{100n}{100} = \frac{945}{100}$$

$$n = 9.45$$

Example 2: What percent of 42 is 21?

$$\frac{n}{100} = \frac{21}{42}$$

$$100 \cdot 21 = n \cdot 42$$

$$2100 = 42n$$

$$\frac{2100}{42} = \frac{42n}{42}$$

$$n = 50\%$$

Example 3: 25 is 40% of what number?

$$\frac{40}{100} = \frac{25}{n}$$

$$100 \cdot 25 = n \cdot 40$$

$$2500 = 40n$$

$$\frac{2500}{40} = \frac{40n}{40}$$

$$n = 62.5$$

APPLICATIONS OF PERCENTS

Example 1 On a 120-question test, a student got 96 correct answers. What percent of the problems did the student work correctly?

The problem states that we have 96 correct answers out of a possible 120.

The problem can be restated as: **96 is what percent of 120?**

$$96 = n \cdot 120$$

$$\frac{96}{120} = \frac{n \cdot 120}{120}$$

$$n = 0.80$$

$$n = 80\%$$

The test score was 80%.

Example 2 How much HCl (hydrochloric acid) is in a 60-milliliter bottle that is marked 80% HCl?

If the bottle is marked 80% HCl, that means that 80% of the solution is HCl and the rest is water. Since the bottle contains 60 milliliters, we can restate the question as: **What is 80% of 60?**

$$n = 0.80 \cdot 60$$

$$n = 48$$

There are 48 milliliters of HCl in 60 milliliters of 80% HCl solution.

Example 3 If 48% of the students in a certain college are female and there are 2,400 female students, what is the total number of students in the college?

We restate the problem as: **2,400 is 48% of what number?**

$$2400 = 0.48 \cdot n$$

$$\frac{2400}{0.48} = \frac{0.48n}{0.48}$$

$$n = 5,000$$

There are 5,000 students.

Example 4 If 25% of the students in elementary algebra courses receive a grade of A, and there are 300 students enrolled in elementary algebra this year, how many students will receive an A?

We restate the problem as: **What number is 25% of 300?**

$$n = 0.25 \cdot 300$$

$$n = 75$$

So, 75 students will receive A's in elementary algebra.

PRACTICE

Write each percent as a fraction with a denominator of 100.

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|--------|--------|--------|--------|
| 1. 20% | 2. 40% | 3. 60% | 4. 80% |
| 5. 24% | 6. 48% | 7. 65% | 8. 35% |

Change each percent to a decimal.

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|-----------|-----------|----------|----------|
| 9. 23% | 10. 34% | 11. 92% | 12. 87% |
| 13. 9% | 14. 7% | 15. 3.4% | 16. 5.8% |
| 17. 6.34% | 18. 7.25% | 19. 0.9% | 20. 0.6% |

Change each decimal to a percent.

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|----------|----------|----------|----------|
| 21. 0.23 | 22. 0.34 | 23. 0.92 | 24. 0.87 |
| 25. 0.45 | 26. 0.54 | 27. 0.03 | 28. 0.04 |
| 29. 0.6 | 30. 0.9 | 31. 0.8 | 32. 0.5 |

Change each percent to a fraction in lowest terms.

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|----------------------|----------------------|-----------------------|-----------------------|
| 33. 4% | 34. 2% | 35. 26.5% | 36. 34.2% |
| 37. 71.87% | 38. 63.6% | 39. 0.75% | 40. 0.45% |
| 41. $6\frac{1}{4}\%$ | 42. $5\frac{1}{4}\%$ | 43. $33\frac{1}{3}\%$ | 44. $66\frac{2}{3}\%$ |

Change each fraction or mixed number to a percent.

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|--------------------|--------------------|--------------------|--------------------|
| 45. $\frac{1}{2}$ | 46. $\frac{1}{4}$ | 47. $\frac{3}{4}$ | 48. $\frac{2}{3}$ |
| 49. $\frac{7}{8}$ | 50. $\frac{1}{8}$ | 51. $\frac{7}{50}$ | 52. $\frac{9}{25}$ |
| 53. $3\frac{1}{4}$ | 54. $2\frac{1}{8}$ | 55. $1\frac{1}{2}$ | 56. $1\frac{3}{4}$ |

57. What number is 25% of 32?
58. What number is 10% of 80?
59. What number is 20% of 120?
60. What number is 15% of 75?
61. What number is 54% of 38?
62. What number is 72% of 200?
63. What number is 11% of 67?
64. What percent of 24 is 12?
65. What percent of 80 is 20?
66. What percent of 50 is 5?
67. What percent of 20 is 4?
68. What percent of 36 is 9?

69. What percent of 70 is 14?
70. What percent of 8 is 6?
71. 32 is 50% of what number?
72. 16 is 20% of what number?
73. 10 is 20% of what number?
74. 11 is 25% of what number?
75. 37 is 4% of what number?
76. 90 is 80% of what number?
77. 8 is 2% of what number?
78. On a 120-question test, a student got 84 correct answers. What percent of the problems did the student work correctly?
79. An engineering student answered 81 questions correctly on a 90-question test. What percent of the questions did she answer correctly? What percent were incorrect?
80. A basketball player made 63 out of 75 free throws. What percent is this?
81. A family spends \$450 every month on food. If the family's income is \$1,800 each month, what percent of the income is spent on food?
82. How much HCl (hydrochloric acid) is in a 60-milliliter bottle that is marked 75% HCl?
83. How much acetic acid is in a 5-liter container that is marked 80% acetic acid? How much is water?
84. A farmer owns 28 acres of land. Of the 28 acres, only 65% can be farmed. How many acres are available for farming?
85. Of the 420 students enrolled in basic math, only 30% are first-year students. How many are first-year students? How many are not?
86. If 48% of the students in a certain college are female and there are 1,440 female students, what is the total number of students in the college?
87. Suppose 60% of the graduating class in a certain high school goes to college. If 240 students from this graduating class are going to college, how many students are in the class?
88. In a shipment of airplane parts, 3% are known to be defective. If 15 parts are found to be defective, how many parts are in the shipment?
89. There are 3,200 students at our school. If 52% of them are men, how many men are enrolled in our school?
90. 75% of the students in chemistry have had algebra. If there are 300 students in chemistry, how many of them have had algebra?

ANSWERS

1. $\frac{20}{100}$
2. $\frac{40}{100}$
3. $\frac{60}{100}$
4. $\frac{80}{100}$
5. $\frac{24}{100}$
6. $\frac{48}{100}$
7. $\frac{65}{100}$
8. $\frac{35}{100}$
9. 0.23
10. 0.34
11. 0.92
12. 0.87
13. 0.09
14. 0.07
15. 0.034
16. 0.058
17. 0.0634
18. 0.0725
19. 0.009
20. 0.006
21. 23%
22. 34%
23. 92%
24. 87%
25. 45%
26. 54%
27. 3%
28. 4%
29. 60%
30. 90%
31. 80%
32. 50%
33. $\frac{1}{25}$
34. $\frac{1}{50}$
35. $\frac{53}{200}$
36. $\frac{171}{500}$
37. $\frac{7187}{10000}$
38. $\frac{159}{250}$
39. $\frac{3}{400}$
40. $\frac{9}{200}$
41. $\frac{1}{16}$
42. $\frac{21}{400}$
43. $\frac{1}{3}$
44. $\frac{2}{3}$
45. 50%
46. 25%
47. 75%
48. $66\frac{2}{3}\%$
49. $87\frac{1}{2}\%$
50. $12\frac{1}{2}\%$
51. 14%
52. 36%
53. 325%
54. $212\frac{1}{2}\%$
55. 150%
56. 175%
57. 8
58. 8
59. 24
60. 11.25
61. 20.52
62. 144
63. 7.37
64. 50%
65. 25%
66. 10%
67. 20%
68. 25%
69. 20%
70. 75%
71. 64
72. 80
73. 50
74. 44
75. 925
76. 112.5
77. 400
78. 70%
79. 90% correctly, 10% incorrectly
80. 84%
81. 25%
82. 45 mi
83. 4 liters acetic acid, 1 liter water
84. 18.2 acres
85. 126 are first-year, 294 are not
86. 3,000 students
87. 400 students
88. 500 parts
89. 1,664 women
90. 225 students